Verification Manual

Authors:

Ferenc Papp

Ph.D. Dr.habil

Associate Professor of Steel Structures

Budapest University of Technology and Economics

József Szalai

Ph.D.

technical director

András Herbay

structural engineer M.Sc

Péter Wálny

structural engineer M.Sc

Consteel Solutions Ltd

CONSTEEL

Content

1. Cross-sections

1.1 Theoretical background

The ConSteel software uses three cross-sectional models:

- Solid Section Model (GSS)
- Elastic Plate Segment Model (EPS)
- Plastic Plate Region Model (PPR)

Cross-sectional properties are computed on these cross-sectional models. The elastic properties given by the GSS model are used in the Analysis module, while the elastic properties given by the EPS and the plastic properties given by the PPR model are used in the Design module of the ConSteel software.

The theoretical background of the GSS model and the computation of the cross-sectional properties are published in the following textbook:

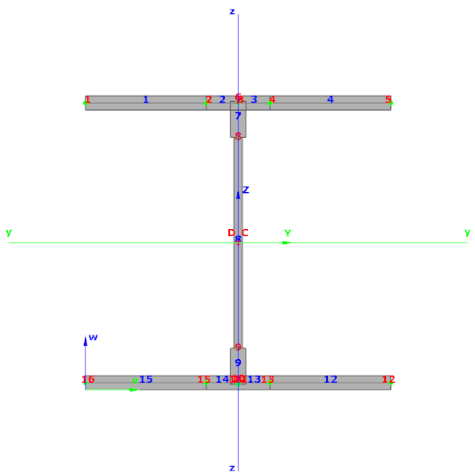
➢ PILKEY, D.W.: Analysis and Design of Elastic Beams: Computational Methods, Wiley, 2002, ISBN:978-0-471-38152-5, pp.153-166 (http://eu.wiley.com/WileyCDA/WileyTitle/productCd 0471381527.html)

The theoretical background of the EPS and PPR models and the computation of the relevant cross-sectional properties are published in the following textbook and article:

➢ KOLBRUNNER, F.C. and BASLER, K: Torsion, Springer, pp. 96-128., Berlin 1966

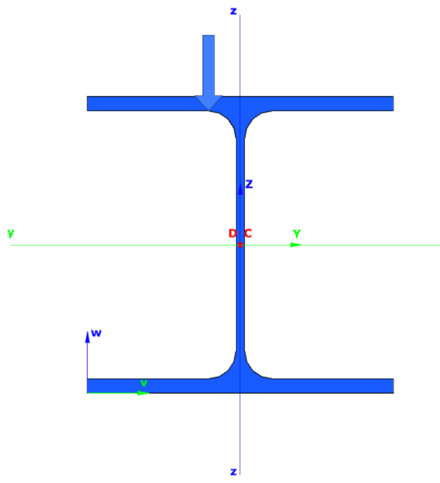
➢ PAPP, F., IVÁNYI, M. and JÁRMAI, K.: Unified object-oriented definition of thin-walled steel beam-column cross-sections, Computers & Structures 79, 839-852, 2001

The EPS model of the HEA300 hot-rolled section is illustrated in the **Figure 1**, the GSS model is illustrated in the **Figure 2**.



Point	v [mm]	w [mm]	t [mm]	Attribute
1	0,0	283,0	14,0	free edge, lower end free (hot-rolled)
2	118,8	283,0	14,0	plate without local buckling attribute
3	150,0	283,0	14,0	plate without local buckling attribute
4	181,3	283,0	14,0	free edge, greater end free (hot-rolled)
5	300,0	283,0	0,0	dummy
6	150,0	283,0	0,0	dummy
7	150,0	284,6	15,2	plate without local buckling attribute
8	150,0	249,0	8,5	web plate
9	150,0	41,0	15,2	plate without local buckling attribute
10	150,0	5,4	0,0	dummy
11	150,0	7,0	0,0	dummy
12	300,0	7,0	14,0	free edge, lower end free (hot-rolled)
13	181,3	7,0	14,0	plate without local buckling attribute
14	150,0	7,0	14,0	plate without local buckling attribute
15	118,8	7,0	14,0	free edge, greater end free (hot-rolled)
16	0,0	7,0		

Fig.1 *EPS model of the HEA300 section*



Point	y [mm]	z [mm]	Y [mm]	Z [mm]
1	-150,0	145,0	-150,0	145,0
2	-150,0	131,0	-150,0	131,0
3	-31,3	131,0	-31,3	131,0
4	-4,3	104,0	-4,3	104,0
5	-4,3	-104,0	-4,3	-104,0
6	-31,3	-131,0	-31,3	-131,0
7	-150,0	-131,0	-150,0	-131,0
8	-150,0	-145,0	-150,0	-145,0
9	150,0	-145,0	150,0	-145,0
10	150,0	-131,0	150,0	-131,0
11	31,3	-131,0	31,3	-131,0
12	4,3	-104,0	4,3	-104,0
13	4,3	104,0	4,3	104,0
14	31,3	131,0	31,3	131,0
15	150,0	131,0	150,0	131,0
16	150,0	145,0	150,0	145,0
17	-20,9	128,9	-20,9	128,9
18	-12,2	123,1	-12,2	123,1
19	-6,3	114,3	-6,3	114,3
20	-6,3	-114,3	-6,3	-114,3
21	-12,2	-123,1	-12,2	-123,1
22	-20,9	-128,9	-20,9	-128,9
23	20,9	-128,9	20,9	-128,9
24	12,2	-123,1	12,2	-123,1
25	6,3	-114,3	6,3	-114,3
26	6,3	114,3	6,3	114,3
27	12,2	123,1	12,2	123,1

Fig.2 *GSS model of the HEA300 section*

1.2 Cross sectional properties

The cross-sectional properties computed by the ConSteel software are checked in the following **Worked Examples** (WE-01 to We-05).

WE-01: Elastic cross-sectional properties of hot rolled sections

Table 1 contains some common hot-rolled sections. The third column of the table shows the elastic cross-sectional properties published in the **Profil ARBED** catalogue. The next columns show the cross-sectional properties computed by the ConSteel software based on both the GSS and the EPS models. The table shows the ratio of the properties given by the catalogue and by the ConSteel software.

Tab.1 Elastic cross-sectional properties of hot rolled sections

section	property	product catalogue [1]	ConSteel			
			GSS [2]	1/2	EPS [3]	1/3
HEA300*	A [mm^2]	11.250	11.311	0.995	11.253	0,999
	I_y [mm^4]	182.600.000	183.495.496	0,995	182.553.772	1,000
	I_z [mm^4]	63.100.000	63.111.171	0,999	63.000.002	1,002
	I_t [mm^4]	851.700	880.686	0,967	851.731	1,000
	I_ω [mm^6]	1,200x10^{12}	1,173x10^{12}	1,023	1,200x10^{12}	1,000
IPE450*	A [mm^2]	98.820	9.917	0,996	9.882	1,000
	I_y [mm^4]	337.400.000	338.882.704	0,996	337.349.907	1,000
	I_z [mm^4]	16.760.000	16.765.473	1,000	16.690.234	1,004
	I_t [mm^4]	668.700	688.277	0,972	668.740	1,000
	I_ω [mm^6]	791,0x10^9	780,2x10^9	1,014	791,0 x10^9	1,000
SHS 150x6,3**	A [mm^2]	3.520	3.475	1,013	3.475	0,987
	I_y [mm^4]	11.900.000	11.688.701	1,018	11.651.937	1,021
	I_z [mm^4]	11.900.000	11.688.701	1,018	11.651.863	1,021
	I_t [mm^4]	19.100.000	19.221.994	0,994	19.144.461	0,998
	I_ω [mm^6]	-	38.710.832	-	0	-

CHS 219,1x6,3**	A [mm^2]	4.210	4.221	**0,997**	4.185	**1,006**
	I_y [mm^4]	23.900.000	23.699.446	**1,008**	23.087.091	**1,035**
	I_z [mm^4]	23.900.000	23.699.383	**1,008**	23.086.742	**1,035**
	I_t [mm^4]	47.700.000	47.398.828	**1,006**	45.572.785	**1,047**
	I_ω [mm^6]	-	1	-	2	-
L 100x12*	A [mm^2]	2.271	2.273	**0,999**	2.256	**1,007**
	I_y [mm^4]	3.280.000	3.270.741	**1,003**	3.322.336	**0,987**
	I_z [mm^4]	854.200	856.647	**0,997**	830.584	**1,028**
	I_t [mm^4]	110.790	120.086	**0,922**	108.277	**1,023**
	I_ω [mm^6]	-	72.790.004	-	0	-

* Profil ARBED, October 1995
** Mannesmann-Stahlbau-Hohlprofile (MSH), Technische Information 1

Evaluation

The GSS model gives accurate results for the elastic cross-sectional properties used in the *Analysis*, see **Figure 3** for case of IPE450 section. The greatest deviations to the values of the Profil ARBED catalogue can be found in the torsional properties, where the maximum deviation is not more than 3,3% in I_t, excepting the L 100x12 section where it is 7,8% (it is mentioned that the I_t of L section does not matter too much in the analysis).
The EPS model is a simplified engineering model which gives approximated values for the elastic cross-sectional properties used in the design, see **Figure 4** for case of IPE450 section. The greatest deviation to the values of the Profil ARBED catalogue is 3,5% in I_y and 4,7% in I_t of the CHS219,1x6,3 section, (it is mentioned that I_t of CHS sections does not matter too much in the design).

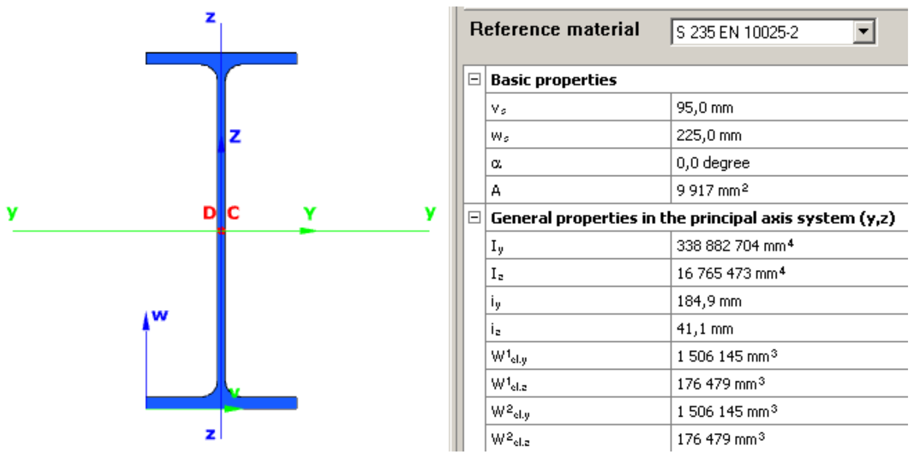
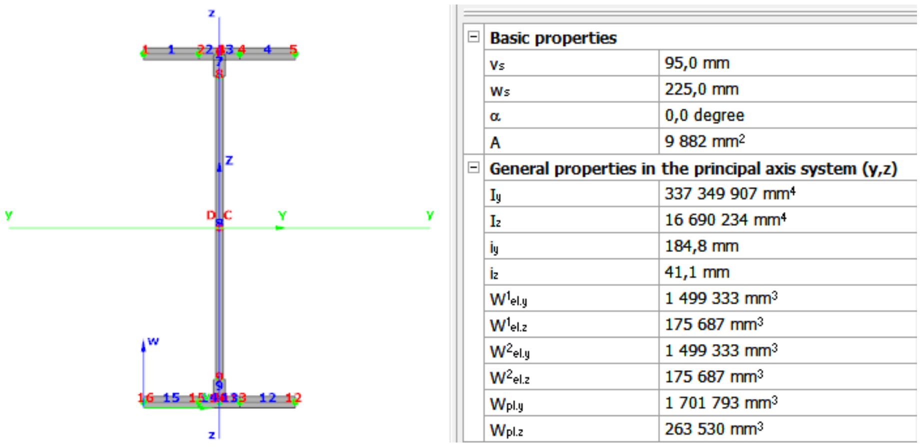


***Fig.3** GSS model and the computed properties of the IPE450 section*



Basic properties	
v_s	95,0 mm
w_s	225,0 mm
α	0,0 degree
A	9 882 mm²
General properties in the principal axis system (y,z)	
I_y	337 349 907 mm⁴
I_z	16 690 234 mm⁴
i_y	184,8 mm
i_z	41,1 mm
$W^1_{el.y}$	1 499 333 mm³
$W^1_{el.z}$	175 687 mm³
$W^2_{el.y}$	1 499 333 mm³
$W^2_{el.z}$	175 687 mm³
$W_{pl.y}$	1 701 793 mm³
$W_{pl.z}$	263 530 mm³

Fig.4 *EPS model and the computed properties of the IPE450 section*

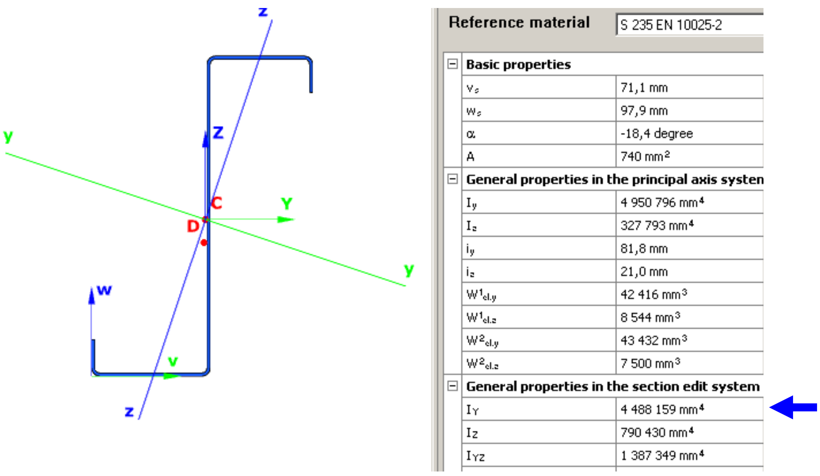
WE-02: Elastic cross-sectional properties of cold formed sections

Table 2 contains some common cold-formed sections. The third column of the table shows the inertia moment about the Y-Y global system given in the **Lindab** catalogue. The next columns show the inertia moment computed by the ConSteel Software based on both GSS and EPS models. The table shows the ratio of the properties given by the catalogue and by the ConSteel Software.

Tab.2 Elastic cross-sectional properties of cold formed sections

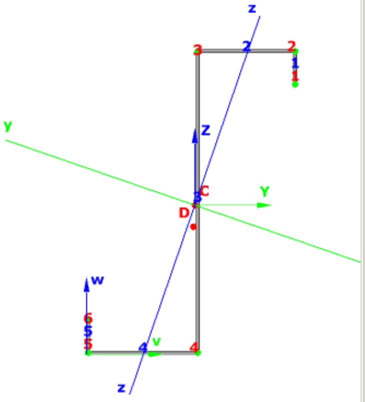
section	property	Lindab catalogue [1]	ConSteel			
			GSS [2]	1/2	EPS [3]	1/3
Lindab Z200* 2 mm	I_Y [mm^4]	4.431.000	4.488.159	**0,987**	4.636.548	**0,956**
Lindab C150* 1,5 mm	I_Y [mm^4]	1.262.000	1.273.452	**0,991**	1.332.359	**0,947**

* Lindab Construline, Technical information - Z-C-U sections (in Hungarian)



Reference material	S 235 EN 10025-2
Basic properties	
v_s	71,1 mm
w_s	97,9 mm
α	-18,4 degree
A	740 mm²
General properties in the principal axis systen	
I_y	4 950 796 mm⁴
I_z	327 793 mm⁴
i_y	81,8 mm
i_z	21,0 mm
$W^1_{el.y}$	42 416 mm³
$W^1_{el.z}$	8 544 mm³
$W^2_{el.y}$	43 432 mm³
$W^2_{el.z}$	7 500 mm³
General properties in the section edit system	
I_Y	4 488 159 mm⁴
I_Z	790 430 mm⁴
I_{YZ}	1 387 349 mm⁴

Fig.5 *GSS model and the computed I_Y property of the Z200-2mm cold formed section*



Basic properties	
v_s	71,1 mm
w_s	97,9 mm
α	-18,5 degree
A	755 mm²
General properties in the principal axis system (y,z)	
I_y	5 118 313 mm⁴
I_z	343 704 mm⁴
i_y	82,3 mm
i_z	21,3 mm
$W^1_{el.y}$	43 470 mm³
$W^1_{el.z}$	7 891 mm³
$W^2_{el.y}$	44 469 mm³
$W^2_{el.z}$	8 986 mm³
$W_{pl.y}$	55 862 mm³
$W_{pl.z}$	13 809 mm³
General properties in the section edit system translated to th	
I_Y	4 636 548 mm⁴
I_Z	825 469 mm⁴
I_{YZ}	1 438 104 mm⁴

Fig.6 *EPS model and the computed I_Y property of the Z200x2mm cold formed section*

Evaluation

The GSS model (see **Figure 5**) provides accurate result for the cold formed cross-sectional property. The EPS model (see **Figure 6**) is a simplified engineering model where the radiuses of the cross-sectional corners are neglected. This approximation results in 5-6% deviation to the exact values.

WE-03: Plastic cross-sectional properties of hot rolled and welded sections

Table 3 contains some common hot rolled and welded sections. The third column of the table shows the plastic cross-sectional modulus given by the **Lindab** catalogue. The next columns show the $W_{pl.y}$ and $W_{pl.z}$ properties computed by the ConSteel software based on the PPR model (which is generated from the EPS model automatically). The last column of the table shows the ratio of the properties given by the catalogue and by the ConSteel software.

Tab.3 Plastic cross-sectional properties of hot rolled and welded sections

section	property	catalogue [1] /theory	ConSteel	
			PPR [2]	1/2
HEA450*	$W_{pl.y}$ [mm^3]	3.216.000	3.215.868	1,000
	$W_{pl.z}$ [mm^3]	965.500	945.000	1,022
IPE450*	$W_{pl.y}$ [mm^3]	1.702.000	1.701.793	1,000
	$W_{pl.z}$ [mm^3]	276.400	263.530	1,049
UAP250*	$W_{pl.y}$ [mm^3]	391.800	384.325	1,019
	$W_{pl.z}$ [mm^3]	87.640	86.303	1,015
	$W_{pl.z}$ [mm^3]	900.000	900.000	1.000
SHS250x6,3***	$W_{pl.y}$ [mm^3]		544.095	
	$W_{pl.z}$ [mm^3]		544.094	
CHS329x6,3***	$W_{pl.y}$ [mm^3]		623.277	
	$W_{pl.z}$ [mm^3]		623.273	
W1** flange: 240-16 web: 400-12	$W_{pl.y}$ [mm^3]	2.077.000	2.077.440	1,000
	$W_{pl.z}$ [mm^3]	460.800	460.800	1.000
W2** flange: 300-20 web: 800-12	$W_{pl.y}$ [mm^3]	6.840.000	6.840.000	1.000

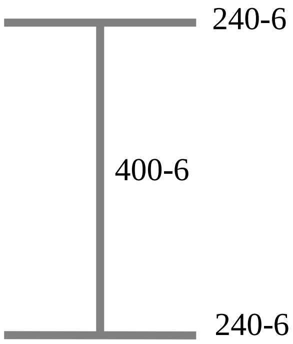
* Profil ARBED, October 1995
** double symmetric welded I section
*** Mannesmann-Stahlbau-Hohlprofile (MSH), Technische Information 1

Evaluation

The PPR model (which is generated from the EPS model automatically) gives approximated numerical result for the plastic cross-sectional modulus of cross-sections. The maximum deviation of the computed values to the exact results is less than 2-3%, excepting the $W_{pl.z}$ property where the effect of the neck area is considerable (for example in case of IPE450 the deviation is 4,9% for the safe).

WE-04: Effective cross-sectional area

Figure 7 shows a double symmetric welded I section (**W4**), which classified to Class 4 due to pure compression. The effective area is calculated by hand using the formulas given by EC3-1-1 and EC3-1-5 and by the ConSteel software.



Grade of material: S275

Size of fillet weld: 3 mm

Fig.7 *Class 4 double symmetric welded I section (**W4**).*

A) Calculation by hand

Section data

flange $b_f := 240 \cdot mm$ $\quad t_f := 6 \cdot mm$

web $h_w := 400 \cdot mm$ $\quad t_w := 6 \cdot mm$

weld $a := 3 \cdot mm$

Design strength

$$f_y := 275 \cdot \frac{N}{mm^2} \qquad \varepsilon := \sqrt{\frac{235 \frac{N}{mm^2}}{f_y}} = 0.924$$

Stress gradient $\Psi := 1.0$

Effective width of web

$$c_w := h_w - 2 \cdot a = 394 \cdot mm$$

$$k_\sigma := 4.0$$

$$\lambda_w := \frac{\frac{c_w}{t_w}}{28.4 \varepsilon \cdot \sqrt{k_\sigma}} = 1.251$$

$$\rho_w := \frac{\left[\lambda_w - 0.055(3 + \Psi)\right]}{\lambda_w^{\,2}} = 0.659$$

$$b_{eff.w} := \rho_w \cdot c_w = 259.622 \cdot mm$$

Effective width of flange

$$c_f := \frac{b_f}{2} - \frac{t_w}{2} - a = 114 \cdot mm$$

$$k_\sigma := 0.43$$

$$\lambda_f := \frac{\frac{c_f}{t_f}}{28.4 \varepsilon \cdot \sqrt{k_\sigma}} = 1.104$$

$$\rho_f := \frac{\left(\lambda_f - 0.188\right)}{\lambda_f^{\,2}} = 0.752$$

$$b_{eff.f} := \rho_f \cdot c_f = 85.698 \cdot mm$$

Effecetive area

$$A_{eff} := \left(b_{eff.w} + 2 \cdot a\right) \cdot t_w + 4 \cdot \left(b_{eff.f} + \frac{t_w}{2} + a\right) \cdot t_f = 3794 \cdot mm^2$$

B) Computation by ConSteel

First by the *Section administration/W4/Properties/Model/Sectional forces* tools a virtual (for example -100 kN) compressive force should be defined, than the effective EPS model and the relevant effective cross-sectional properties can be available, see **Figure 8**.

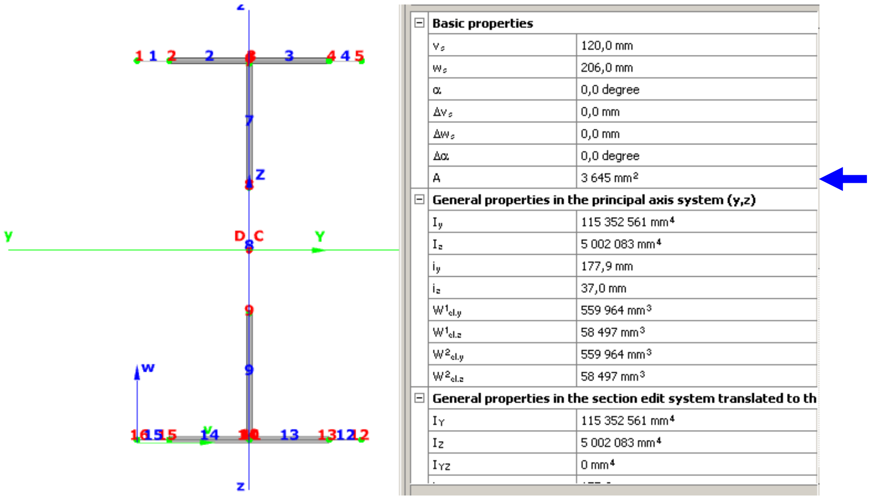


Fig.8 Effective EPS model of the W4 section due to pure compression

Evaluation

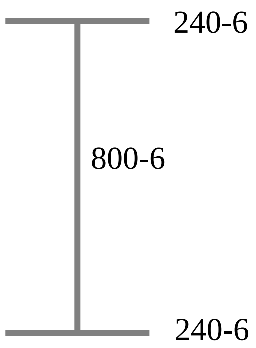
Table 4 shows the effective cross-sectional areas of the W4 welded I section calculated by hand using the formulas of EC3-1-1 and EC3-1-5 and by the ConSteel software. The deviation is 4% for the safe (the effective EPS model neglects the web thickness and the size of the weld in the calculation of the basic plate width).

Tab.4 Effective cross-sectional area of welded I section

section	property	theory [1]	EPS [2]	1/2
W4	A_{eff} [mm^2]	3.794	3.645	**1,040**

WE-05: Effective cross-sectional modulus

Figure 9 shows a double symmetric welded I section (**W5**), which classified to Class 4 due to bending about the major and the minor axes. The effective sectional modulus is calculated by hand using the formulas of EC3-1-1 and EC3-1-5 and by the ConSteel software.



Grade of material: S275

Size of fillet weld: 3 mm

*Fig.9 Class 4 double symmetric welded I section (**W5**)*

A) Calculation by hand

Bending about major axis

Section data

flange $b_f := 240 \cdot mm$ $\quad t_f := 6 \cdot mm$

web $h_w := 800 \cdot mm$ $\quad t_w := 6 \cdot mm$

weld $a := 3 \cdot mm$

Design strength $f_y := 275 \cdot \frac{N}{mm^2}$ $\quad \varepsilon := \sqrt{\frac{235 \frac{N}{mm^2}}{f_y}} = 0.924$

Effective width of flange $c_f := \frac{b_f}{2} - \frac{t_w}{2} - a = 114 \cdot mm$

$k_\sigma := 0.43$

$$\lambda_f := \frac{\frac{c_f}{t_f}}{28.4\varepsilon \cdot \sqrt{k_\sigma}} = 1.104$$

$$\rho_f := \frac{(\lambda_f - 0.188)}{\lambda_f^2} = 0.752$$

$b_{eff.f} := \rho_f \cdot c_f = 85.698 \cdot mm$

Working width $b_{w.f} := 2b_{eff.f} + t_w + a = 180.4 \cdot mm$

Effective width of web using iterative procedure

Step 1

Centroid of section $A_1 := (b_{w.f} + b_f) \cdot t_f + h_w \cdot t_w = 7322.4 \cdot mm^2$

$$S_{Y.1} := (b_f \cdot t_f - b_{w.f} \cdot t_f) \cdot \left(\frac{h_w}{2} + \frac{t_f}{2}\right) = 144123 \cdot mm^3$$

$$Z_{S.1} := \frac{S_{Y.1}}{A_1} = 19.683 \cdot mm$$

Stress gradient in web

$$\Psi := -\frac{\frac{h_w}{2} - a - Z_{S.1}}{\frac{h_w}{2} - a + Z_{S.1}} = -0.906$$

Effctiv e width of web

$$b_w := h_w - 2 \cdot a = 794 \cdot mm$$

$$k_\sigma := 7.81 - 6.29\Psi + 9.78\Psi^2 = 21.525$$

$$\lambda_w := \frac{\frac{b_w}{t_w}}{28.4\varepsilon \cdot \sqrt{k_\sigma}} = 1.086$$

$$\rho_w := \frac{\left[\lambda_w - 0.055(3 + \Psi)\right]}{\lambda_w^{\ 2}} = 0.823$$

$$b_c := \frac{b_w}{2} + Z_{S.1} = 416.683 \cdot mm$$

$$b_{eff.w} := \rho_w \cdot \frac{b_w}{1 - \Psi} = 342.861 \cdot mm$$

$$b_{e1} := 0.4 b_{eff.w} = 137.1 \cdot mm$$

$$b_{e2} := 0.6 b_{eff.w} = 205.7 \cdot mm$$

$$b_1 := b_{e1} + a = 140.1 \cdot mm$$

$$b_0 := b_c - b_{e1} - b_{e2} = 73.821 \cdot mm$$

$$b_2 := h_w - \left(b_c + a\right) + b_{e2} = 586.034 \cdot mm$$

Step 2

Centroid of section

$$A_2 := A_1 - b_0 \cdot t_w = 6879.4 \cdot mm^2$$

$$S_{Y.2} := S_{Y.1} + b_0 \cdot t_w \cdot \left(\frac{h_w}{2} - b_1 - \frac{b_0}{2}\right) = 242872 \cdot mm^3$$

$$Z_S := \frac{S_{Y.2}}{A_2} = 35.304 \cdot mm$$

Stress gradient in web

$$\Psi := -\frac{\frac{h_w}{2} - a - Z_S}{\frac{h_w}{2} - a + Z_S} = -0.837$$

Effctiv e width of web

$$k_\sigma := 7.81 - 6.29\Psi + 9.78\Psi^2 = 19.919$$

$$\lambda_w := \frac{\frac{b_w}{t_w}}{28.4\varepsilon \cdot \sqrt{k_\sigma}} = 1.129$$

$$\rho_w := \frac{\left[\lambda_w - 0.055(3 + \Psi)\right]}{\lambda_w^{\ 2}} = 0.792$$

$$b_c := \frac{b_w}{2} + Z_S = 432.304 \cdot mm$$

$$b_{eff.w} := \rho_w \cdot \frac{b_w}{1 - \Psi} = 342.445 \cdot mm$$

$$b_{e1} := 0.4 b_{eff.w} = 137 \cdot mm$$

$$b_{e2} := 0.6 b_{eff.w} = 205.5 \cdot mm$$

$$b_1 := b_{e1} + a = 140 \cdot mm$$

$$b_0 := b_c - b_{e1} - b_{e2} = 89.9 \cdot mm$$

$$b_2 := h_w - \left(b_c + a\right) + b_{e2} = 570.2 \cdot mm$$

Step 3

Centroid of section

$$A_3 := A_1 - b_0 \cdot t_w = 6783.222\,mm^2$$

$$S_{Y.3} := S_{Y.1} + b_0 \cdot t_w \cdot \left(\frac{h_w}{2} - b_1 - \frac{b_0}{2} \right) = 260091 \cdot mm^3$$

$$Z_S := \frac{S_{Y.3}}{A_3} = 38.343 \cdot mm$$

Stress gradient in web

$$\Psi := -\frac{\frac{h_w}{2} - a - Z_S}{\frac{h_w}{2} - a + Z_S} = -0.824$$

Effctive width of web

$$k_\sigma := 7.81 - 6.29\Psi + 9.78\Psi^2 = 19.63$$

$$\lambda_w := \frac{\frac{b_w}{t_w}}{28.4\varepsilon \cdot \sqrt{k_\sigma}} = 1.138$$

$$\rho_w := \frac{\left[\lambda_w - 0.055(3 + \Psi)\right]}{\lambda_w^{\;2}} = 0.787$$

$$b_c := \frac{b_w}{2} + Z_S = 435.3 \cdot mm$$

$$b_{eff.w} := \rho_w \cdot \frac{b_w}{1 - \Psi} = 342.4 \cdot mm$$

$$b_{e1} := 0.4 b_{eff.w} = 137 \cdot mm$$

$$b_{e2} := 0.6 b_{eff.w} = 205.4 \cdot mm$$

$$b_1 := b_{e1} + a = 140 \cdot mm$$

$$b_0 := b_c - b_{e1} - b_{e2} = 92.94 \cdot mm$$

$$b_2 := h_w - \left(b_c + a\right) + b_{e2} = 567.1 \cdot mm$$

Inertia moment about y-y axis

$$h_1 := \frac{h_w}{2} + Z_S = 438.343 \cdot mm$$

$$h_2 := h_w - h_1 = 361.657 \cdot mm$$

$$I_1 := b_{w.f} \cdot t_f \cdot \left(h_1 + \frac{t_f}{2} \right)^2 = 210829061 \cdot mm^4$$

$$I_2 := b_f \cdot t_f \cdot \left(h_2 + \frac{t_f}{2} \right)^2 = 191483328 \cdot mm^4$$

$$I_3 := \frac{t_w \cdot b_1^{\;3}}{12} + b_1 \cdot t_w \cdot \left(h_1 - \frac{b_1}{2} \right)^2 = 115318910 \cdot mm^4$$

$$I_4 := \frac{t_w \cdot b_2^{\;3}}{12} + b_2 \cdot t_w \cdot \left(h_2 - \frac{b_2}{2} \right)^2 = 111947503 \cdot mm^4$$

$$I_{eff.y} := I_1 + I_2 + I_3 + I_4 = 629578802 \cdot mm^4$$

Sectional moduli

$$W_{eff.y1} := \frac{I_{eff.y}}{h_1 + t_f} = 1416875 \cdot mm^3$$

$$W_{eff.y2} := \frac{I_{eff.y}}{h_2 + t_f} = 1712409 \cdot mm^3$$

Bending about minor axis

Effective width of flange

$$c_f := \frac{b_f}{2} - \frac{t_w}{2} - a = 114 \cdot mm$$

$$\Psi := 0 \qquad k_\sigma := 0.57$$

$$\lambda_f := \frac{\frac{c_f}{t_f}}{28.4\varepsilon \cdot \sqrt{k_\sigma}} = 0.959$$

$$\rho_f := \frac{(\lambda_f - 0.188)}{\lambda_f^2} = 0.839$$

$$b_{eff.f} := \rho_f \cdot c_f = 95.601 \cdot mm$$

Working width

$$b_{w.f} := \frac{b_f}{2} + b_{eff.f} + \frac{t_w}{2} + a = 221.6 \cdot mm$$

Effective width of web using iterative procedure

Step 1

Centroid of section

$$A_1 := 2b_{w.f} \cdot t_f + h_w \cdot t_w = 7459.2 \cdot mm^2$$

$$S_{Y.1} := 2 \cdot b_{w.f} \cdot t_f \cdot \frac{(b_f - b_{w.f})}{2} = 24462.9 \cdot mm^3$$

$$Y_{S.1} := \frac{S_{Y.1}}{A_1} = 3.28 \cdot mm$$

Effctive width of web

$$b_w := h_w - 2 \cdot a = 794 \cdot mm$$

$$\Psi := 1.0 \qquad k_\sigma := 4.0$$

$$\lambda_w := \frac{\frac{b_w}{t_w}}{28.4\varepsilon \cdot \sqrt{k_\sigma}} = 2.52$$

$$\rho_w := \frac{[\lambda_w - 0.055(3 + \Psi)]}{\lambda_w^2} = 0.362$$

$$b_{eff.w} := \rho_w \cdot b_w = 287.5 \cdot mm$$

$$b_{e1} := 0.5 b_{eff.w} = 143.8 \cdot mm$$

$$b_{e2} := 0.5 b_{eff.w} = 143.8 \cdot mm$$

$$b_{w.w} := b_{eff.w} + 2 \cdot a = 293.5 \cdot mm$$

Step 2

Centroid of section

$$A_2 := 2b_{w.f} \cdot t_f + (b_{eff.w} + 2 \cdot a) \cdot t_w = 4420.5 \cdot mm^2$$

$$S_{Y.2} := 2 \cdot b_{w.f} \cdot t_f \cdot \frac{(b_f - b_{w.f})}{2} = 24462.9 \cdot mm^3$$

$$Y_{S.2} := \frac{S_{Y.2}}{A_2} = 5.534 \cdot mm$$

Stress gradient in flange

$$\Psi := \frac{Y_{S.2} + \frac{t_w}{2} + a}{\frac{b_f}{2} + Y_{S.2}} = 0.092$$

Effective width of flange

$$k_\sigma := 0.57 - 0.21 \cdot \Psi + 0.07 \cdot \Psi^2 = 0.551$$

$$\lambda_f := \frac{\frac{c_f}{t_f}}{28.4\varepsilon \cdot \sqrt{k_\sigma}} = 0.975$$

$$\rho_f := \frac{(\lambda_f - 0.188)}{\lambda_f^2} = 0.828$$

$$b_{eff.f} := \rho_f \cdot c_f = 94.4 \cdot mm$$

$$b_{w.f} := \frac{b_f}{2} + b_{eff.f} + \frac{t_w}{2} + a = 220.4 \cdot mm$$

Inertia moment about z-z axis

$$I_1 := b_{w.w} \cdot t_w \cdot Y_{S.2}^{\,2} = 53939 \cdot mm^4$$

$$I_2 := 2 \cdot \left(t_f \cdot \frac{b_{w.f}^{\,3}}{12} + b_{w.f} \cdot t_f \cdot Y_{S.2}^{\,2} \right) = 10787084\ mm^4$$

$$I_{eff.z} := I_1 + I_2 = 10841023\ mm^4$$

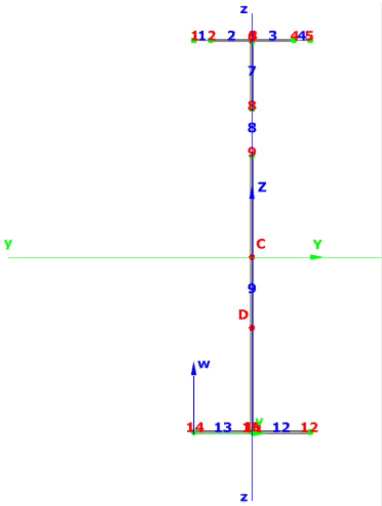
Sectional moduli

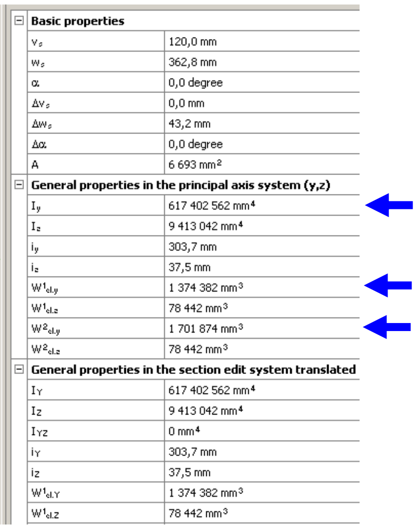
$$W_{eff.z1} := \frac{I_{eff.z}}{\frac{b_f}{2} - Y_{S.2}} = 94710 \cdot mm^3$$

$$W_{eff.z2} := \frac{I_{eff.z}}{b_{w.f} - \frac{b_f}{2} + Y_{S.2}} = 102338 \cdot mm^3$$

B) Computation by ConSteel

First by the *Section administration/W5/Properties/Model/Sectional forces* tools a virtual bending moment (for example M_y=-100 kNm than M_z=100 kNm) should be defined, than the effective EPS model and the relevant effective cross-sectional properties can be available, see **Figure 10**.





Basic properties	
v_s	120,0 mm
w_s	362,8 mm
α	0,0 degree
Δv_s	0,0 mm
Δw_s	43,2 mm
Δα	0,0 degree
A	6 693 mm²
General properties in the principal axis system (y,z)	
I_y	617 402 562 mm⁴
I_z	9 413 042 mm⁴
i_y	303,7 mm
i_z	37,5 mm
$W^1_{el.y}$	1 374 382 mm³
$W^1_{el.z}$	78 442 mm³
$W^2_{el.y}$	1 701 874 mm³
$W^2_{el.z}$	78 442 mm³
General properties in the section edit system translated	
I_Y	617 402 562 mm⁴
I_Z	9 413 042 mm⁴
I_{YZ}	0 mm⁴
i_Y	303,7 mm
i_Z	37,5 mm
$W^1_{el.Y}$	1 374 382 mm³
$W^1_{el.Z}$	78 442 mm³

Fig.10 *Effective EPS model of the W5 section due to bending about major axis*

Evaluation

Table 5 shows the effective inertia moment and sectional modulus of the **W5** welded I section computed by the theoretical formulas of EC3-1-1 and EC3-1-5 and by the ConSteel software. The results are accurate, the maximum deviation in sectional modulus is 2,9% for the safe (the effective EPS model neglects the web thickness and the size of the weld in calculation the basic plate width, but uses iterative procedure).

Tab.5 Effective cross-sectional modulus of welded I section

section	property	theory [1]	effective EPS [2]	1/2
W5	$\mathbf{I_{eff.y}}$ [mm^4]	6,296 x 10^8	6,174 x 10^8	**1,020**
	$\mathbf{W_{eff.y1}}$ [mm^3]	1.414.875	1.374.382	**1,029**
	$\mathbf{W_{eff.y2}}$ [mm^3]	1.712.409	1.701.874	**1,006**
	$\mathbf{W_{eff.z1}}$ [mm^3]	94.710	94.602	**1,001**
	$\mathbf{W_{eff.z2}}$ [mm^3]	102.338	101.580	**1,007**

1.3 Elastic stresses

Elastic stresses of sections computed by the ConSteel software are checked in the following **Worked Examples** (WE-06 and WE-07).

WE-06: Elastic stresses in hot rolled section

Elastic stresses in the HEA300 hot-rolled section are calculated by hand using the theoretical formulas and computed by the ConSteel software.

A) Calculation by hand

Section: HEA300

Properties from Profil ARBED catalogue

$A := 11250 \cdot mm^2$ $\quad t_w := 8.5 \cdot mm$

$I_y := 182600000 \cdot mm^4$ $\quad W_{el.y} := 1260000 \cdot mm^3$

$S_y := 692088 \cdot mm^3$ (by EPSmodel)

$I_\omega := 1200000000000 \, mm^6$

Compression $\quad N_x := 400 \cdot kN \quad \sigma_N := \frac{N_x}{A} = 35.56 \frac{N}{mm^2}$

Bending $\quad M_y := 240 \cdot kN \cdot m \quad \sigma_{My} := \frac{M_y}{W_{el.y}} = 190.5 \frac{N}{mm^2}$

Shear $\quad V_z := 220 \cdot kN \quad \tau_{z.max} := \frac{V_z \cdot S_y}{I_y \cdot t_w} = 98.1 \cdot \frac{N}{mm^2}$

Warping $\quad B := 5 \cdot kN \cdot m^2 \quad \omega := 20700 \cdot mm^2$ (by EPS)

$\sigma_\omega := \frac{B}{I_\omega} \cdot \omega = 86.25 \frac{N}{mm^2}$

Interaction of pure cases $\quad \sigma_{x.max} := \sigma_N + \sigma_{My} + \sigma_\omega = 312.3 \frac{N}{mm^2}$

A) Computation by ConSteel

The stress are visualized in the Section module, see **Figure 11**.

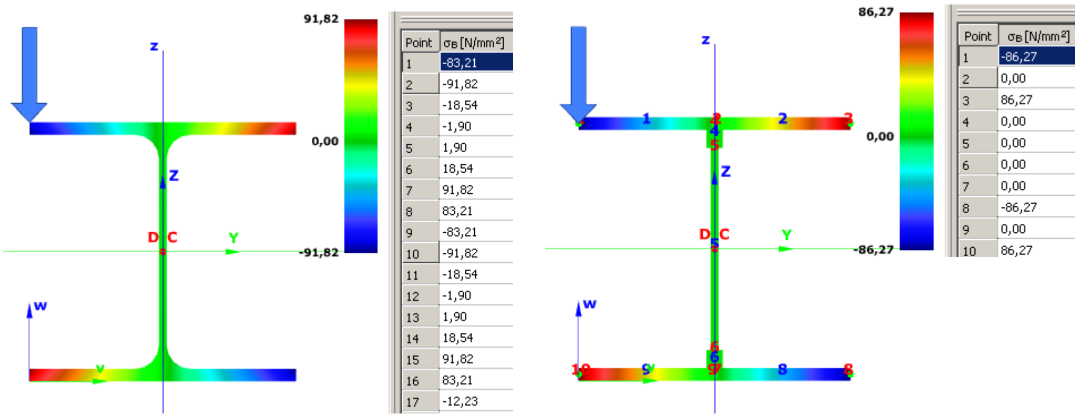


Fig.11 Elastic stresses in the HEA300 section by the GSS and the EPS model

Evaluation

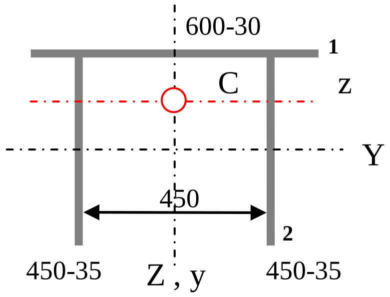
Table 6 shows the stress components in the HEA300 cross-section calculated by hand using theoretical formulas and by the ConSteel software using the GSS and EPS cross-sectional models. The GSS model may be the accurate in warping stress since it takes the change of the stresses through the thickness of the plates into consideration. The EPS model gives 5,0% deviation in bending stress to the theoretical result (stresses visualized in Analysis module are calculated in the counter line of the plates, but in the Design module they are calculated in the extreme fibers, see value in brackets).

Tab.6 Elastic stresses in hot rolled section

section	stress component [N/mm²]	theory [1]	ConSteel			
			GSS [2]	1/2	EPS [3]	1/3
HEA300	σ_N	35,56	35,36	1.006	35,55	1.000
	σ_{My}	190,5	189,6	1.005	181,43 (190,4)	1.050 1.000
	σ_ω	86,25	83,21	1.037	86,27	0,999
	σ_x	312,3	308,2	1.013	303,24	1.030

WE-07: Elastic stresses in welded section

Figure 12 shows a symmetric welded hat section (**W7**), which classified to Class 3 due to the both compression and bending about the major axis. The elastic stresses are calculated by hand using the theoretical formulas and by the ConSteel software.



Grade of material: S235

Size of fillet weld: 8 mm

Internal forces: N_X = -1200kN
M_Y = -360 kNm

***Fig.12 W7** welded symmetric hat section*

A) Calculation by hand

Section data

flange $b_f := 600 \cdot mm$ $t_f := 30 \cdot mm$

webs $h_w := 450 \cdot mm$ $t_w := 35 \cdot mm$ $b_w := 450 \cdot mm$

weld $a := 8 \cdot mm$

Material $f_y := 275 \cdot \frac{N}{mm^2}$ $\varepsilon := \sqrt{\frac{235 \frac{N}{mm^2}}{f_y}} = 0.924$

Gross Area $A := b_f \cdot t_f + 2 \cdot h_w \cdot t_w = 49500 \cdot mm^2$

Centroid $S_Y := b_f \cdot t_f \cdot \left(\frac{h_w}{2} + \frac{t_f}{2} \right) = 4320000 \cdot mm^3$

$Z_C := \frac{S_Y}{A} = 87.273 \cdot mm$

$z_{comp} := \frac{h_w}{2} + Z_C = 312.273 \cdot mm$

Class of section

- pure compression

flange $c_f := b_w - 2 \cdot a = 434 \cdot mm$

$\eta := \frac{c_f}{t_f} = 14.467 \quad < \quad 33 \cdot \varepsilon = 30.506$ Class 1

web $c_w := h_w - a = 442 \cdot mm$

$\eta := \frac{c_w}{t_w} = 12.629 \quad > \quad 10 \cdot \varepsilon = 9.244$
$\quad < \quad 14 \cdot \varepsilon = 12.94$ Class 3

- pure bending about major axis

web $c_w := h_w - a = 442 \cdot mm$

$\alpha := \frac{z_{comp}}{c_w} = 0.706$

$\Psi := -\frac{c_w - Z_C}{c_w + Z_C} = -0.670$

$k_\sigma := 0.57 - 0.21 \cdot \Psi + 0.07 \cdot \Psi^2 = 0.742$

$\eta := \frac{c_w}{t_w} = 12.629 \quad < \quad 21 \cdot \varepsilon \cdot \sqrt{k_\sigma} = 16.724$ Class 3

$$\alpha_{plastic} := \frac{2 \cdot h_w + b_f \cdot \frac{t_f}{t_w}}{4 \cdot c_w} = 0.800$$

$$\eta := \frac{c_w}{t_w} = 12.629 \quad > \quad 10 \cdot \frac{\varepsilon}{\alpha_{plastic}} = 11.556 \qquad \text{Class 3}$$

Elastic sectional modulus about major axis

$$I_z := b_f \cdot t_f \cdot \left(\frac{h_w}{2} + \frac{t_f}{2} - Z_C \right)^2 + 2 \cdot t_w \cdot \frac{h_w^{\,3}}{12} + 2 \cdot h_w \cdot t_w \cdot Z_C^{\,2} = 1191344318\ \text{mm}^4$$

$$z_1 := \frac{h_w}{2} + t_f - Z_C = 167.727 \cdot \text{mm} \qquad z_2 := \frac{h_w}{2} + Z_C = 312.273 \cdot \text{mm}$$

$$W_{el.z.1} := \frac{I_z}{z_1} = 7102866 \cdot \text{mm}^3 \qquad W_{el.z.2} := \frac{I_z}{z_2} = 3815076 \cdot \text{mm}^3$$

Elastic stresses

Normal force $\quad N_X := -1200 \cdot \text{kN}$

Bending moment $\quad M_Y := 360 \cdot \text{kN} \cdot \text{m}$

$$\sigma_{x.1} := \frac{N_X}{A} - \frac{M_Y}{W_{el.z.1}} = -74.9 \cdot \frac{\text{N}}{\text{mm}^2}$$

$$\sigma_{x.2} := \frac{N_X}{A} + \frac{M_Y}{W_{el.z.2}} = 70.1 \cdot \frac{\text{N}}{\text{mm}^2}$$

B) Computation by ConSteel

The stress are visualized in the Section module, see **Figure 13**.

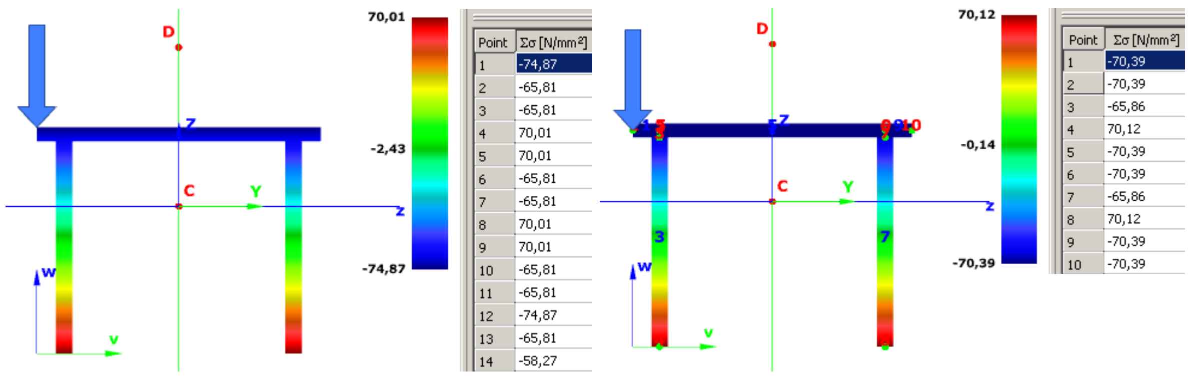


Fig.13 *Elastic stresses in the W7 welded hat section by the GSS and the EPS model*

Evaluation

Table 7 shows the stress components in the **W7** welded hat section calculated by hand using theoretical formulas and by the ConSteel software using the GSS and EPS cross-sectional models. The GSS model is accurate. The EPS model gives 5,1% deviation in bending stress to the theoretical result (stresses visualized in Analysis module are calculated in the counter line of the plates, but in the Design module they are calculated in the extreme fibers, see value in brackets).

Tab.7 Elastic stresses in welded hat section

section	property	dimension	theory [1]	ConSteel			
				GSS [2]	1/2	EPS [3]	1/3
W7	A	mm^2	49.500	49.500	1.000	49.500	1.000
	Class of flange		1			1	
	Class of web - compression - bending		 1 3			 1 3	
	I_z	$x10^6\ mm^4$	1.191	1.193	0.998	1.191	1.000
	$W_{el.1}$	$x10^3\ mm^3$	7.103	7.111	0.999	7.103	1.000
	$W_{el.2}$	$x10^3\ mm^3$	3.815	3.819	0.999	3.815	1.000
	$\sigma_{x.1}$	N/mm^2	-74,9	-74.9	1.000	70.4 (74,9)	1.064 1.000
	$\sigma_{x.2}$	N/mm^2	70,1	70,0	1.001	70.1	1.000

1.4 Design resistances

Cross sectional design resistances are calculated by hand using the rules of EC3-1-1 and by the ConSteel software in the following **Worked Examples** (WE-08 to WE-19).

WE-08: Compression (Class 1 section)

The design resistance for pure compression of the **HEA300** hot-rolled section is calculated by hand and by the ConSteel software.

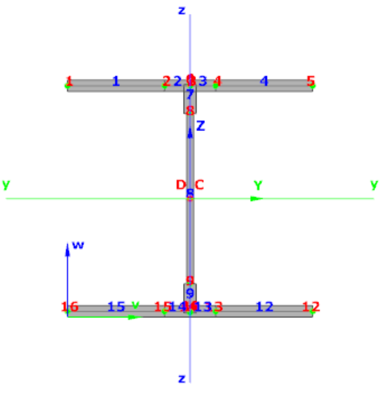
A) Calculation by hand

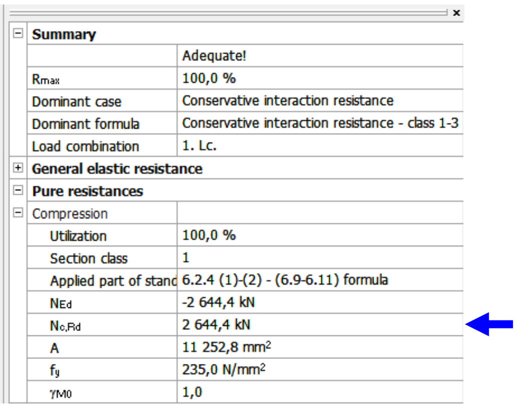
HEA300 section

Class of section	Class 1
Grade of material	S235
	$f_y := 235 \cdot \frac{N}{mm^2}$
Gross area	$A := 11250 \cdot mm^2$
Partial factor	$\gamma_{M0} := 1.0$
Resistance	$N_{pl.Rd} := \frac{A \cdot f_y}{\gamma_{M0}} = 2643.8 \cdot kN$

A) Computation by ConSteel

The computation of the design resistance of the HEA300 section duo to pure compression is shown in **Figure 14**.





Summary	
	Adequate!
R_{max}	100,0 %
Dominant case	Conservative interaction resistance
Dominant formula	Conservative interaction resistance - class 1-3
Load combination	1. Lc.
General elastic resistance	
Pure resistances	
Compression	
Utilization	100,0 %
Section class	1
Applied part of stand	6.2.4 (1)-(2) - (6.9-6.11) formula
N_{Ed}	-2 644,4 kN
$N_{c,Rd}$	2 644,4 kN
A	11 252,8 mm²
f_y	235,0 N/mm²
γ_{M0}	1,0

Fig.14 Design resistance of HEA300 section for compression

Evaluation

Table 8 shows the design resistance of the HEA300 section for pure compressive force computed by hand and by the ConSteel software. The result of the ConSteel software is accurate.

Tab.8 Cross-sectional resistance of HEA300 section for compression

section	compressive resistance [kN]		
	theory [1]	ConSteel (EPS model) [2]	1/2
HEA300	2.644	2.644	**1,000**

WE-09: Compression (Class 4 section)

The design resistance of the welded **W4** section (see WE-04) for pure compression is calculated by hand and by the ConSteel software.

A) Calculation by hand

Class of section — Class 4

Grade of material — S275

$$f_y := 275 \cdot \frac{N}{mm^2}$$

Effective area — $A_{eff} := 3794 \cdot mm^2$ (see WE – 04)

Partial factor — $\gamma_{M0} := 1.0$

Resistance — $$N_{pl.Rd} := \frac{A_{eff} \cdot f_y}{\gamma_{M0}} = 1043.3\ kN$$

A) Computation by ConSteel

The computation of the design resistance of the **W4** welded Class 4 section duo to pure compression is shown in **Figure 15**.

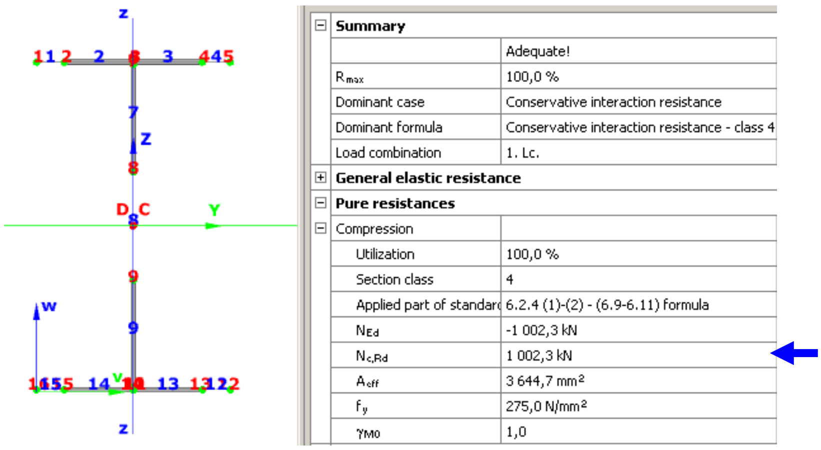


Fig.15 Design resistance of the W4 welded Class 4 section for compression

Evaluation

Table 9 shows the design resistance of the **W4** welded Class 4 section for compressive force computed by hand and by the ConSteel software. The result deviates to the safe (effective EPS model takes the total width of plate for the basic width).

Tab.9 Cross-sectional resistance of W4 welded section for compression

section	compressive resistance [kN]		
	theory [1]	ConSteel (EPS model) [2]	1/2
W4	1043,3	1002,3	**1,041**

WE-10: Bending about major axis (Class 1 section)

The design resistance of the IPE450 hot-rolled I section for bending about major axis is calculated by hand and by the ConSteel software.

A) Calculation by hand

Class of section — Class 1

Grade of material — S235

$$f_y := 235 \cdot \frac{N}{mm^2}$$

Plastic modulus — $W_{pl.y} := 1702000 \cdot mm^3$ (see WE – 03)

Partial factor — $\gamma_{M0} := 1.0$

Resistance — $$M_{pl.y.Rd} := \frac{W_{pl.y} \cdot f_y}{\gamma_{M0}} = 400.0 kN \cdot m$$

A) Computation by ConSteel

The computation of the design resistance of the **IPE450** hot-rolled Class 1 section duo to pure bending about major axis is shown in **Figure 16**.

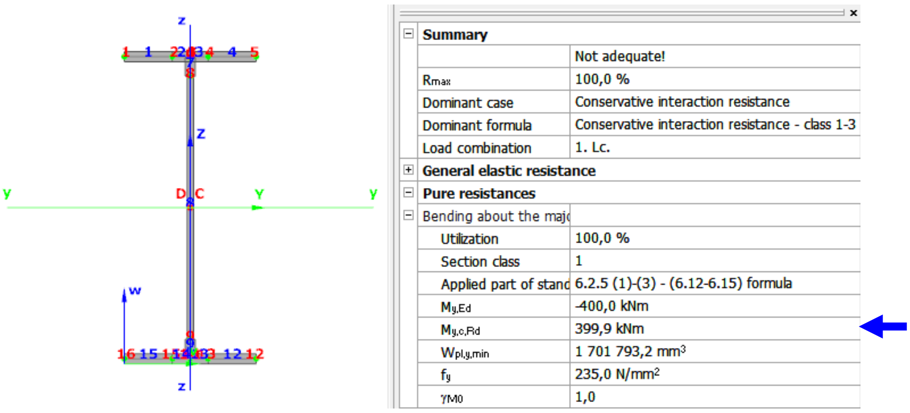


Fig.16 Design resistance of the IPE450 Class 1 section for bending about major axis

Evaluation

Table 10 shows the cross-sectional resistance of the IPE450 section for pure bending about major axis calculated by hand and by the ConSteel software. The result is accurate.

Tab.10 Cross-sectional resistance of IPE450 section for bending about major axis

section	bending resistance about major axis [kNm]		
	theory [1]	ConSteel (EPS model) [2]	1/2
IPE450	400,0	399,9	**1,000**

WE-11: Bending about minor axis (Class 1 section)

The design resistance of the **HEA450** hot-rolled I section for bending about minor axis is calculated by hand and by the ConSteel software.

A) Calculation by hand

Class of section Class 1

Grade of material S235

$$f_y := 235 \cdot \frac{N}{mm^2}$$

Plastic modulus $W_{pl.z} := 965500 \cdot mm^3$ (see WE – 03)

Partial factor $\gamma_{M0} := 1.0$

Resistance $$M_{pl.z.Rd} := \frac{W_{pl.z} \cdot f_y}{\gamma_{M0}} = 226.9 kN \cdot m$$

B) Computation by ConSteel

The computation of the design resistance of the **HEA450** hot-rolled Class 1 section duo to pure bending about minor axis is shown in **Figure 17**.

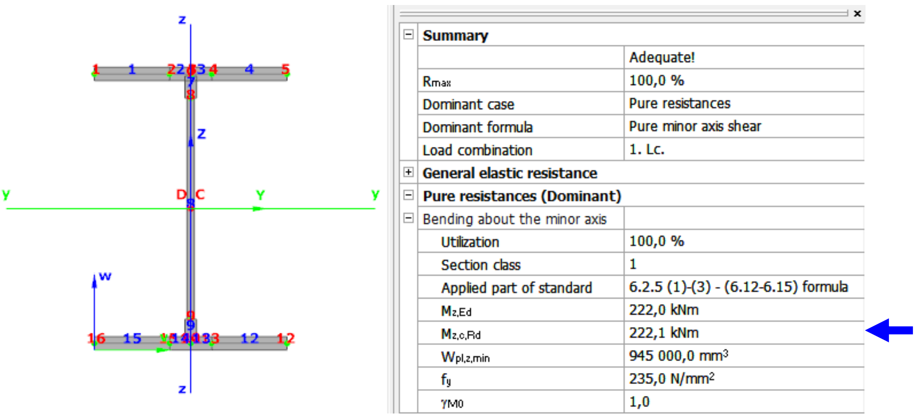


Fig.17 Design resistance of the HEA450 Class 1 section for bending about minor axis

Evaluation

Table 11 shows the cross-sectional resistance of the HEA450 section for pure bending about minor axis calculated by theory and by the ConSteel software. The result is accurate (EPS model takes the effect of the neck area approximately).

Tab.11 Cross-sectional resistance of HEA450 section for bending about minor axis

section	bending resistance about minor axis [kNm]		
	theory [1]	ConSteel (EPS model) [2]	1/2
HEA450	226,9	222,1	1,022

WE-12: Bending about major axis (Class 4 section)

The design resistance of the welded Class 1 **W5** section (see WE-04) for pure bending major axis is calculated by hand and by the ConSteel software.

A) Calculation by hand

Class of section: Class 4

Grade of material: S275

$$f_y := 275 \cdot \frac{N}{mm^2}$$

Effective modulus: $W_{eff.y} := 1416875 \cdot mm^3$ (see WE – 05)

Partial factor: $\gamma_{M0} := 1.0$

Resistance: $M_{eff.y.Rd} := \frac{W_{eff.y} \cdot f_y}{\gamma_{M0}} = 389.6kN \cdot m$

B) Computation by ConSteel

The computation of the design resistance of the **W5** welded section duo to pure bending about minor axis is shown in **Figure 18**.

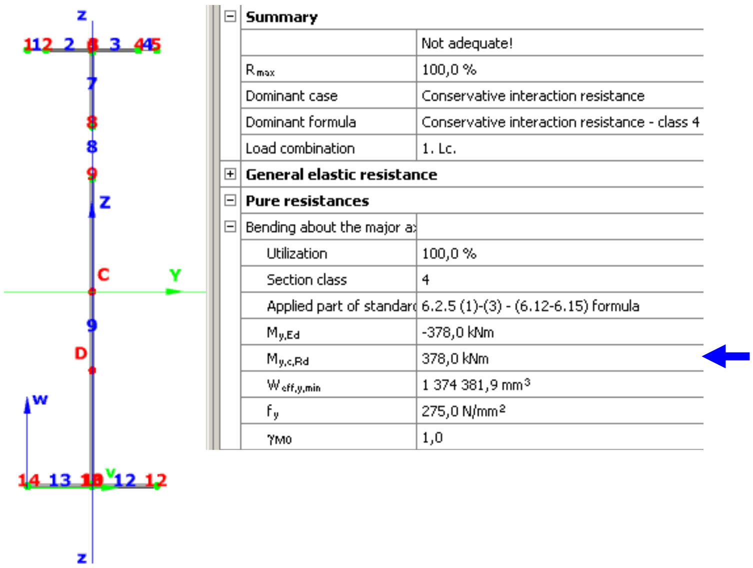


Fig.18 Design resistance of the W5 welded Class 4 section for bending about major axis

Evaluation

Table 12 shows the cross-sectional resistance of the **W5** welded Class 4 section for pure bending about major axis calculated by hand using the simplified rules of EC3-1-1 and EC3-1-5 and by the ConSteel software. The result is accurate for the safe (effective EPS model computes the effective cross-section by the iterative procedure proposed by EC3-1-5).

Tab.12 Cross-sectional resistance of the W5 welded Class 4 section for bending about major axis

section	bending resistance about major axis [kNm]		
	theory [1]	ConSteel (EPS model) [2]	1/2
W5	389,6	378,0	**1,031**

1) simplified method with no iteration

WE-13: Bending about minor axis (Class 4 section)

The design resistance of the welded **W5** Class 1 section (see WE-04) for pure bending about minor axis is calculated by hand and by the ConSteel software.

A) Calculation by hand

Class of section	Class 4
Grade of material	S275
	$f_y := 275 \cdot \frac{N}{mm^2}$
Effective modulus	$W_{eff.z} := 94710 \cdot mm^3$ (see WE – 05)
Partial factor	$\gamma_{M0} := 1.0$
Resistance	$M_{eff.y.Rd} := \frac{W_{eff.z} \cdot f_y}{\gamma_{M0}} = 26.045\,kN \cdot m$

B) Computation by ConSteel

The computation of the design resistance of the Class 1 **W5** welded section duo to pure bending about minor axis is shown in **Figure 19**.

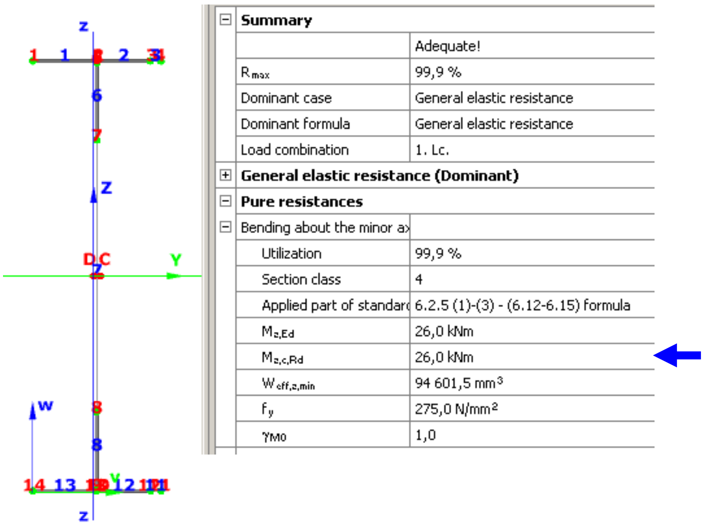


Fig.19 Design resistance of the W5 welded Class 4 section for bending about minor axis

Evaluation

Table 13 shows the cross-sectional resistance of the W5 welded Class 4 section for pure bending about minor axis calculated by the simplified rules of the EC3-1-1 and EC3-1-5 and by the ConSteel software. The result is accurate.

Tab.13 Cross-sectional resistance of the W5 welded section for bending about minor axis

section	bending resistance about minor axis [kNm]		
	theory [1]	ConSteel (EPS model) [2]	1/2
W5	26,045	26,0	1,002

1) with one iteration step

WE-14: Shear of web (Class 1 section)

The design resistance of the **IPE450** section (see WE-04) for shear in the direction of minor axis is calculated by hand and by the ConSteel software.

A) Calculation by hand

Class of section	Plastic
Grade of material	S235
	$f_y := 235 \cdot \dfrac{N}{mm^2}$
Shear area	$A_{vz} := 5085 \cdot mm^2$ (ProfilARBED)
Partial factor	$\gamma_{M0} := 1.0$
Resistance	$V_{pl.Rd} := \dfrac{A_{vz} \cdot f_y}{\gamma_{M0} \cdot \sqrt{3}} = 689.9 \cdot kN$

B) Computation by ConSteel

The computation of the shear design resistance of the IPE450 section is shown in **Figure 20**.

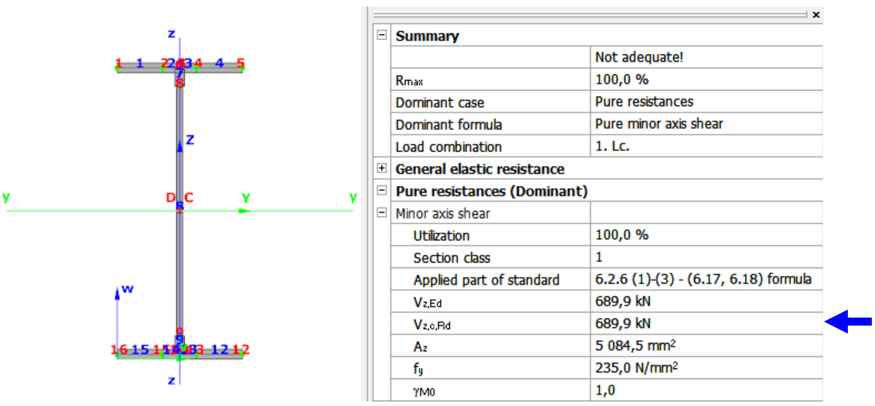


Fig.20 Design shear resistance of the IPE450 section

Evaluation

Table 14 shows the shear cross-sectional resistance of the IPE450 section computed by the ConSteel software. The result is accurate.

Tab.14 Cross-sectional resistance of IPE450 section for web shear

section	shear resistance of web [kN]		
	theory [1]	ConSteel (EPS model) [2]	1/2
IPE450	689,9	689,9	1,000

WE-15: Bending with shear effect (Class 1 section)

The design bending resistance about the major axis of the **IPE450** section (see WE-04) with shear effect is calculated by hand and by the ConSteel software.

A) Calculation by hand

Design shear force $V_{z.Ed} := 500 \cdot kN$

Shear resistance $V_{pl.Rd} = 689.9 \cdot kN$ (see WE – 14)

Reduction factor $\rho := \left(\frac{2 \cdot V_{z.Ed}}{V_{pl.Rd}} - 1 \right)^2 = 0.202$

Web area $d := 378.8 \cdot mm$ $t_w := 9.4 \cdot mm$

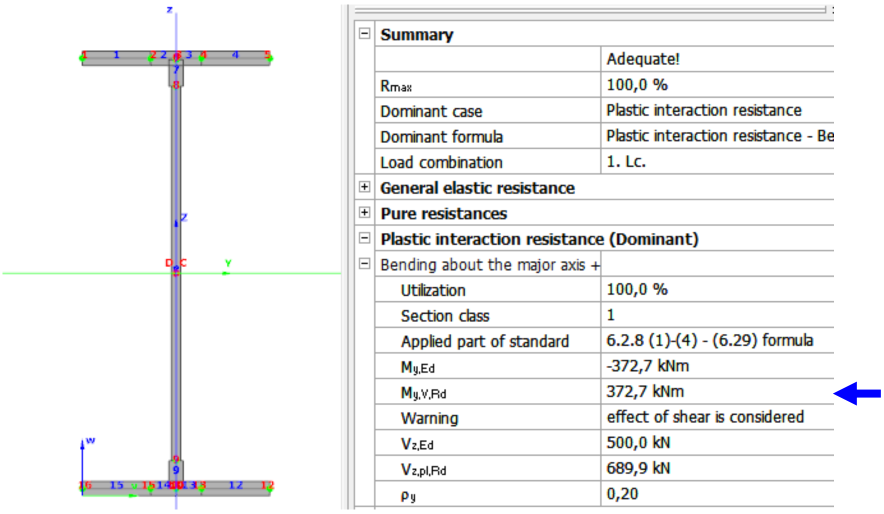
$A_w := d \cdot t_w = 3560.7 \cdot mm^2$

Sectional moduli $W_{pl.y} := 1702000 \cdot mm^3$

Resistance $M_{y.V.Rd} := \frac{\left(W_{pl.y} - \frac{\rho \cdot A_w^{\ 2}}{4 \cdot t_w} \right) \cdot f_y}{\gamma_{M0}} = 384.0 kN \cdot m$

B) Computation by ConSteel

The computation of the design resistance of the IPE450 section is shown in **Figure 21**.



Summary	
	Adequate!
R_{max}	100,0 %
Dominant case	Plastic interaction resistance
Dominant formula	Plastic interaction resistance - Be
Load combination	1. Lc.
General elastic resistance	
Pure resistances	
Plastic interaction resistance (Dominant)	
Bending about the major axis +	
Utilization	100,0 %
Section class	1
Applied part of standard	6.2.8 (1)-(4) - (6.29) formula
$M_{y,Ed}$	-372,7 kNm
$M_{y,V,Rd}$	372,7 kNm
Warning	effect of shear is considered
$V_{z,Ed}$	500,0 kN
$V_{z,pl,Rd}$	689,9 kN
ρ_y	0,20

Fig.21 Design bending resistance with shear effect of the IPE450 section

Evaluation

Table 15 shows the design bending resistance of the IPE450 section with shear effect calculated by hand and by the ConSteel software. The result is accurate.

Tab.15 Cross-sectional resistance of IPE450 section for bending with shear effect

section	bending resistance with shear effect [kNm]		
	theory [1]	ConSteel (EPS model) [2]	1/2
IPE450	384,0	372,7	**1,03**

WE-16: Bending and Axial Force (Class 1 section)

The design bending resistance about the major axis of the **HEA450** section (see WE-11) with axial force effect is calculated by hand and by the ConSteel software.

A) Calculation by hand

Design axial forces $N_{Ed} := -1600 \cdot kN$

Properties $A := 17800 \cdot mm^2$ (ProfileARBED)

$W_{pl.y} := 3216000 \cdot mm^3$

Flange data $b_f := 300 \cdot mm$ $t_f := 21 \cdot mm$

Grade of material S235

$$f_y := 235 \cdot \frac{N}{mm^2}$$

Comressive resistance $N_{pl.Rd} := \frac{A \cdot f_y}{\gamma_{M0}} = 4183.0 kN$

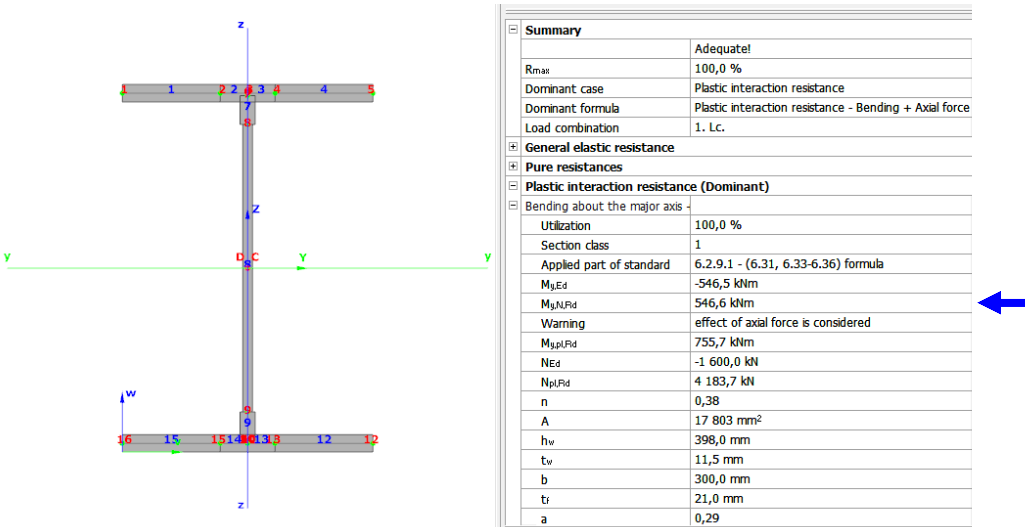
Parameters $n := \frac{-N_{Ed}}{N_{pl.Rd}} = 0.383$ $a := \frac{(A - 2 \cdot b_f \cdot t_f)}{A} = 0.292$

Resistance $M_{pl.y.Rd} := \frac{W_{pl.y} \cdot f_y}{\gamma_{M0}} = 755.76 kN \cdot m$

$$M_{N.y.Rd} := M_{pl.y.Rd} \frac{(1-n)}{(1-0.5a)} = 546.5 kN \cdot m$$

B) Computation by ConSteel

The computation of the design resistance of the HEA450 section is shown in **Figure 22**.



Summary	
	Adequate!
Rmax	100,0 %
Dominant case	Plastic interaction resistance
Dominant formula	Plastic interaction resistance - Bending + Axial force
Load combination	1. Lc.
General elastic resistance	
Pure resistances	
Plastic interaction resistance (Dominant)	
Bending about the major axis -	
Utilization	100,0 %
Section class	1
Applied part of standard	6.2.9.1 - (6.31, 6.33-6.36) formula
My,Ed	-546,5 kNm
My,N,Rd	546,6 kNm
Warning	effect of axial force is considered
My,pl,Rd	755,7 kNm
NEd	-1 600,0 kN
Npl,Rd	4 183,7 kN
n	0,38
A	17 803 mm²
hw	398,0 mm
tw	11,5 mm
b	300,0 mm
tf	21,0 mm
a	0,29

Fig.22 Design bending resistance of the HEA450 section with axial force effect

Evaluation

Table 16 shows the bending resistance of the HEA450 section with axial force effect calculated by hand and by the ConSteel software. The result is accurate.

Tab.16 Design bending resistance of HEA450 section with axial force effect

section	bending resistance with axial force effect [kNm]		
	theory [1]	ConSteel (EPS model) [2]	1/2
HEA450	546,5	546,6	**1.000**

WE-17: Bending and Axial Force (Class 3 section)

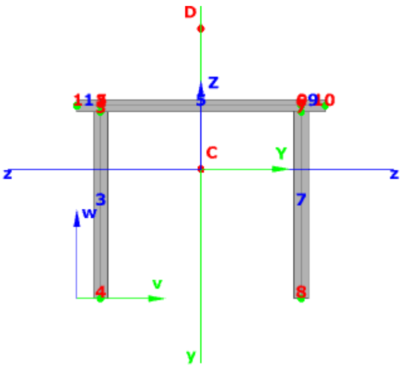
The design bending resistance about the major axis of the **W7** welded hat section (see WE-07) with axial force effect is calculated by hand and by the ConSteel software.

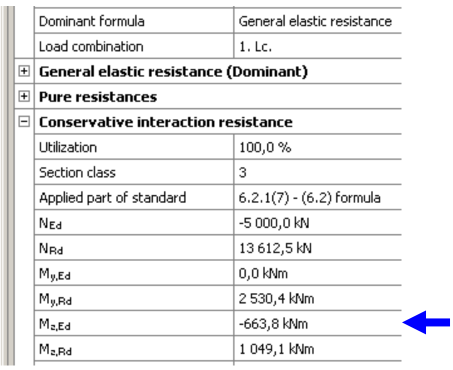
A) Calculation by hand

Design compressive forces	$N_{Ed} := 5000 \cdot kN$
Class of section	Class 3
Grade of material	S275
	$f_y := 275 \cdot \frac{N}{mm^2}$
Sectional properties	$A := 49500 \cdot mm^2$
	$W_{el.z.min} := 3815000 \cdot mm^3$
Bending resistance	$M_{y.Rd} := \left(1 - \frac{N_{Ed}}{A \cdot \frac{f_y}{\gamma_{M0}}}\right) \cdot W_{el.z.min} \cdot \frac{f_y}{\gamma_{M0}} = 663.8 kN \cdot m$

B) Computation by ConSteel

The computation of the design resistance of the W7 welded hat section is shown in **Figure 23**.





Dominant formula	General elastic resistance
Load combination	1. Lc.
General elastic resistance (Dominant)	
Pure resistances	
Conservative interaction resistance	
Utilization	100,0 %
Section class	3
Applied part of standard	6.2.1(7) - (6.2) formula
N_{Ed}	-5 000,0 kN
N_{Rd}	13 612,5 kN
$M_{y,Ed}$	0,0 kNm
$M_{y,Rd}$	2 530,4 kNm
$M_{z,Ed}$	-663,8 kNm
$M_{z,Rd}$	1 049,1 kNm

Fig.23 Design bending resistance of the W7 welded hat section with axial force effect

Evaluation

Table 17 shows the bending resistance of the W7 welded hat section with axial force effect calculated by hand and by the ConSteel software. The result is accurate.

Tab.17 Design bending resistance of HEA450 section with axial force effect

section	bending resistance with axial force effect [kNm]		
	theory [1]	ConSteel (EPS model) [2]	1/2
HEA450	663,8	663,8	**1,000**

WE-18: Bending and Axial Force (Class 4 section)

The design bending resistance about the major axis of the **W5** welded I section (see WE-05) with axial force effect is calculated by hand and by the ConSteel software.

A) Calculation by hand

Section data

flange $b_f := 240 \cdot mm$ $\quad t_f := 6 \cdot mm$

web $h_w := 800 \cdot mm$ $\quad t_w := 6 \cdot mm$

weld $a := 3 \cdot mm$

Design strength $f_y := 275 \cdot \frac{N}{mm^2}$ $\quad \varepsilon := \sqrt{\frac{235 \frac{N}{mm^2}}{f_y}} = 0.924$

Compression

Design compressive force $N_{Ed} := 300 \cdot kN$

Stress gradient $\Psi := 1.0$

Effective width of web $c_w := h_w - 2 \cdot a = 794 \cdot mm$

$k_\sigma := 4.0$

$$\lambda_w := \frac{\frac{c_w}{t_w}}{28.4 \varepsilon \cdot \sqrt{k_\sigma}} = 2.52$$

$$\rho_w := \frac{\left[\lambda_w - 0.055(3 + \Psi)\right]}{\lambda_w^2} = 0.362$$

$b_{eff.w} := \rho_w \cdot c_w = 287.541 \cdot mm$

Effective width of flange $c_f := \frac{b_f}{2} - \frac{t_w}{2} - a = 114 \cdot mm$

$k_\sigma := 0.43$

$$\lambda_f := \frac{\frac{c_f}{t_f}}{28.4 \varepsilon \cdot \sqrt{k_\sigma}} = 1.104$$

$$\rho_f := \frac{\left(\lambda_f - 0.188\right)}{\lambda_f^2} = 0.752$$

$b_{eff.f} := \rho_f \cdot c_f = 85.698 \cdot mm$

Effecetive area

$$A_{eff} := \left(b_{eff.w} + 2 \cdot a\right) \cdot t_w + 4 \cdot \left(b_{eff.f} + \frac{t_w}{2} + a\right) \cdot t_f = 3962 \cdot mm^2$$

Bending about major axis

Sectional moduli (see WE-05)

$W_{eff.y.min} := 1416875 \cdot mm^3$

Resistance

$$M_{y.N.Rd} := \left(1 - \frac{N_{Ed}}{A_{eff} \cdot \frac{f_y}{\gamma_{M0}}}\right) \cdot W_{eff.y.min} \cdot \frac{f_y}{\gamma_{M0}} = 282.4 kN \cdot m$$

B) Computation by ConSteel

The computation of the design bending resistance of the W5 welded hat section is shown in **Figure 24**.

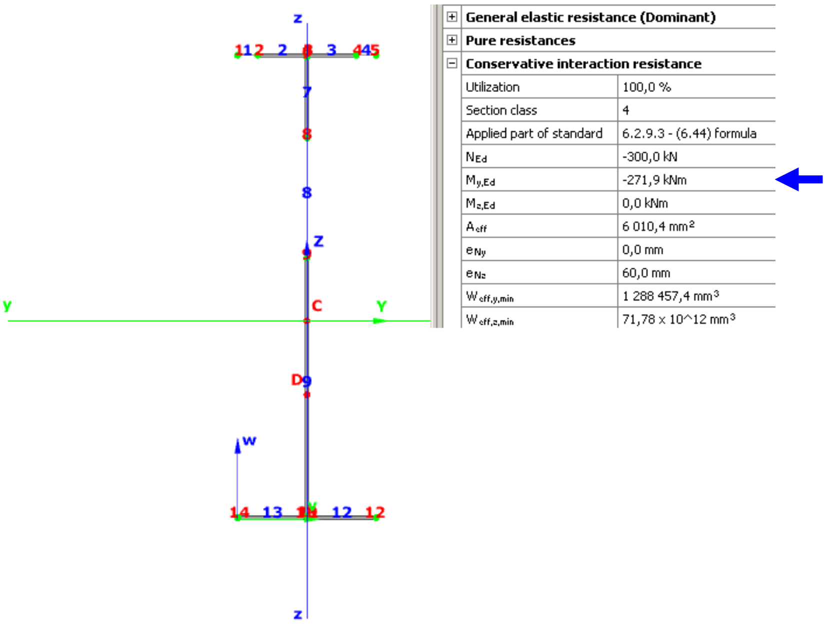


Fig.24 Design bending resistance of the W5 welded I section with axial force effect

Evaluation

Table 18 shows the bending resistance of the W5 welded I section with axial force effect calculated by hand and by the ConSteel software. The hand calculation (theory[1]) used the conservative interaction formula where the effective cross-sectional properties were calculated due to pure compression (A_{eff}) and due to the pure bending moment ($W_{eff.y.min}$). The ConSteel computation used the integrated normal stress distribution due to compression and bending when the effective cross-sectional properties were calculated by iterative procedure. The differences in the cross-sectional properties are considerable, respectively. **However, the difference in the final result (bending resistance) is less than 4%.** ConSteel software gives a more accurate result.

Tab.18 Design bending resistance of the W5 welded I section with axial force effect (Class 4)

section	property	bending resistance with axial force effect		
		theory [1]	ConSteel (eff.EPS model) [2]	1/2
W5	A_{eff} [mm^2]	3.962	6.010	0,659
	$W_{eff.y.min}$ [mm^3]	1.416.875	1.288.458	1.099
	$M_{y.N.Rd}$ [kNm]	282,4	271,9	1.039

WE-19: Biaxial bending with compression force effect (Class 1 section)

The design bending resistance about the major axis of the **HEB400** hot-rolled H section with axial force effect is calculated by hand and by the ConSteel software.

A) Calculation by hand

Properties (Profil ARBED)

Class of section: Class 1

Dimensions: $b := 300 \cdot mm$ $\quad t_f := 24 \cdot mm$

Sectional modulus:

$A := 19780 \cdot mm^2$

$W_{pl.y} := 3232000 \cdot mm^3$

$W_{pl.z} := 1104000 \cdot mm^3$

Design strength $\quad f_y := 235 \cdot \dfrac{N}{mm^2}$

Design forces

Compression: $N_{Ed} := 3000 \cdot kN$

Bending about minor axis: $M_{z.Ed} := 100 \cdot kN \cdot m$

Pure resistances

Compression: $N_{pl.Rd} := \dfrac{A \cdot f_y}{\gamma_{M0}} = 4648.3 \cdot kN$

Parameters: $n := \dfrac{N_{Ed}}{N_{pl.Rd}} = 0.645 \qquad a := \dfrac{A - 2 \cdot b \cdot t_f}{A} = 0.272$

Bending about major axis: $M_{pl.y.Rd} := \dfrac{W_{pl.y} \cdot f_y}{\gamma_{M0}} = 759.52 \cdot kN \cdot m$

$$M_{N.pl.y.Rd} := M_{pl.y.Rd} \frac{1-n}{1-0.5a} = 311.721 \cdot kN \cdot m$$

Bending about minor axis: $M_{pl.z.Rd} := \dfrac{W_{pl.z} \cdot f_y}{\gamma_{M0}} = 259.44 \cdot kN \cdot m$

$$M_{N.pl.z.Rd} := M_{pl.z.Rd} \left[1 - \left(\frac{n-a}{1-a}\right)^2\right] = 191.186 \cdot kN \cdot m$$

Bending resistance about major axis due to biaxial bending with axial force

Parameters: $\alpha := 2 \qquad \beta := 5 \cdot n = 3.227$

$$M_{y.Rd} := M_{N.pl.y.Rd} \sqrt[\alpha]{1 - \left(\frac{M_{z.Ed}}{M_{N.pl.z.Rd}}\right)^{\beta}} = 291.8 \cdot kN \cdot m$$

B) Computation by ConSteel

The computation of the design bending resistance of the HEB400 hot-rolled H section is shown in **Figure 25**.

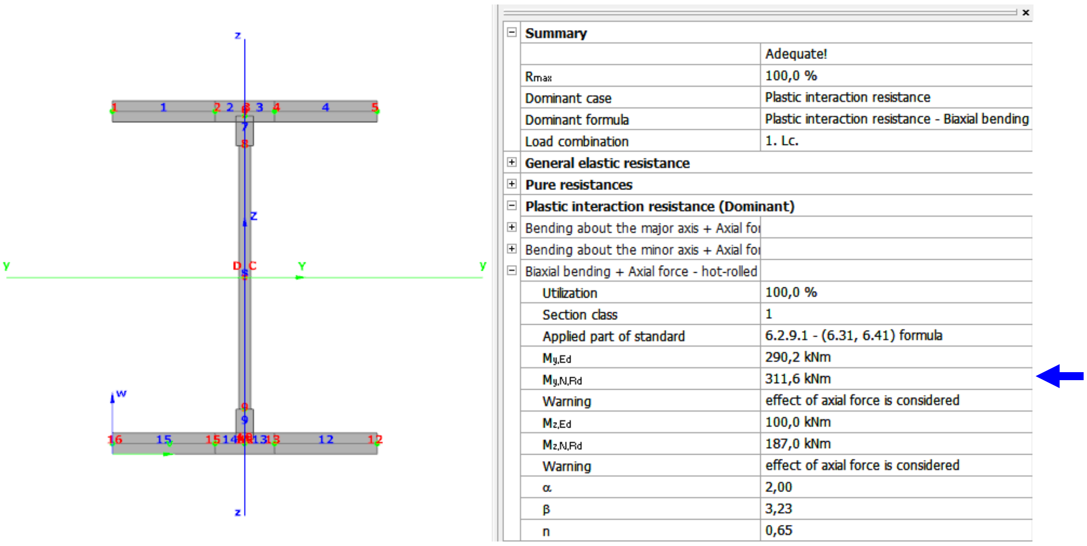


Fig.25 Design bending resistance of the HEB400 section with axial force effect

Evaluation

Table 19 shows the bending resistance of the HEB400 hot-rolled H section with axial force effect calculated by hand and by the ConSteel software. The result is accurate.

Tab.19 Design biaxial bending resistance of the HEB400 section with axial force effect

section	resistance*	bending resistance with axial force effect		
	[kNm]	theory [1]	ConSteel [2]	1/2
HEB400	$M_{N.pl.y.Rd}$	311,7	311,6	1,000
	$M_{N.pl.z.Rd}$	191,2	187,0	1.022
	$M_{y.Rd}$	291,8	290,2	1.005

*) N_{Ed}=-3000 kN ; $M_{z.Ed}$=100 kNm

2. Analysis

2.1 Theoretical background

The ConSteel software uses the 14 degrees of freedom general thin-walled beam-column finite element (referred as **csBeam7**) published by Rajasekaran in the following textbook:

- CHEN, W.F. ATSUTA, T.: Theory of Beam-Columns: Space behavior and design, Vol.2 McGraw-Hill, 1977, pp. 539-564

Later more researchers used and developed this element, for example:

- PAPP, F.: Computer aided design of steel beam-column structures, Doctoral thesis, Budapest University of Technology & Heriot-Watt University of Edinburgh, 1994-1996

The general beam-column finite element takes the effect of warping into consideration, therefore it is reasonable to use it in both of the geometrically nonlinear stress analysis and the elastic stability analysis of spatial steel structures.

The ConSteel software uses a triangular isoparametric thick plane shell finite element with 3 nodes (referred as **csShell3**). The application and the efficiency of this element is discussed in the following papers:

- HRABOK, M.M., HRUDEY, T.M. "A review and catalogue of plate bending finite elements" Computers and Structures. Vol.19. pp.479-495. 1984.
- HENRY, T.Y., SAIGAL, S., MASUD, A., KAPANIA, R.K., "A survey of recent finite elements" International Journal of Numerical Methods in Engineering. Vol. 47. pp.101-127. 2000.

This element may be integrated with the general beam-column finite element sufficiently in a mixed beam-column and plated steel structural model.

2.2 Stress analysis

The stress analysis (computation of deflections, internal forces and reactions) of simple structural members are verified by

- Geometrically linear (first order) theory
- Geometrically non-linear (second order) theory

2.2.1 Geometrically linear (first order) theory

The analysis of simple structural members using the ConSteel software (based on the **csBeam7** and the **csShell3** finite element) are checked in the following **Worked Examples** (**WE-20** to **WE-23**).

WE-20 Compressed member

Figure 26 shows a compressed member. The moving of the end of the member and the compressive stress are calculated by hand and by the ConSteel software using both of the **csBeam7** and the **csSheel3** finite element models.

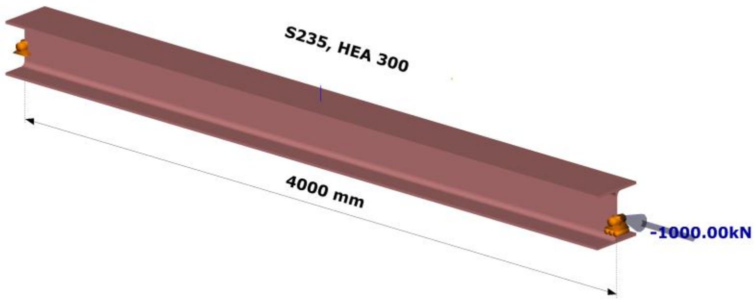


Fig.26 *Stress analysis of compressed member*

A) Calculation by hand

Sectional area	$A := 11250 \cdot mm^2$
Grade of material	S235
	$E := 210000 \cdot \frac{N}{mm^2}$
Length of member	$L := 4000 \cdot mm$
Compressive force	$F_x := 1000 \cdot kN$
Compressive stress	$\sigma_x := \frac{F_x}{A} = 88.889 \cdot \frac{N}{mm^2}$
End moving	$e_x := \sigma_x \cdot \frac{L}{E} = 1.693 \cdot mm$

A) Computation by ConSteel

- Beam-column FE model (csBeam7)

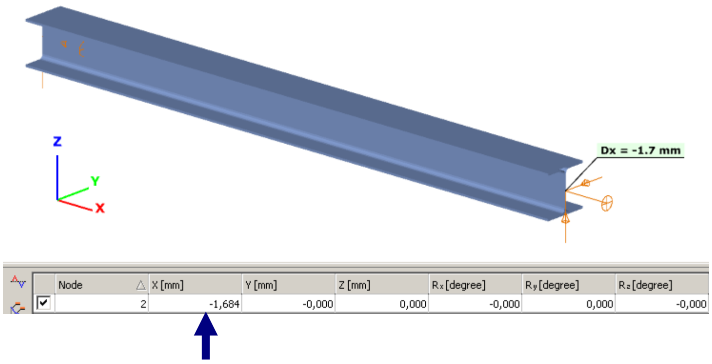


Fig.27 *Axial deflection of the compressed member*

- Shell FE model (csShell3)

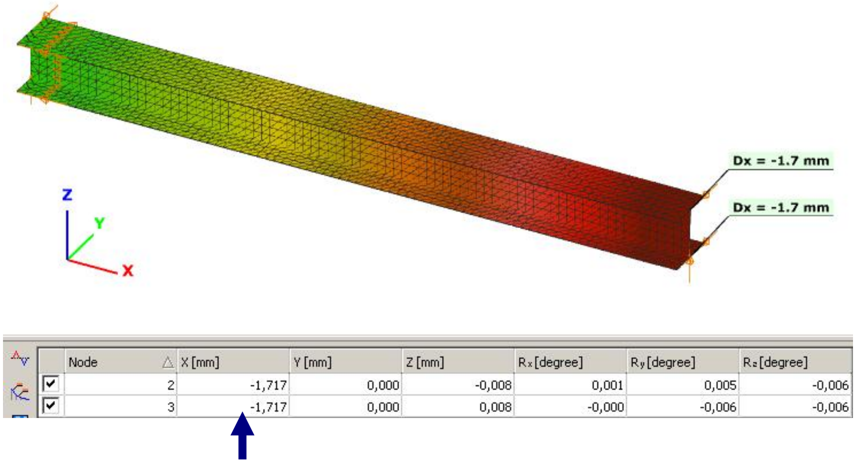


Fig.28 *Axial deflection of the compressed member*

Evaluation

Table 20 shows the axial deflection of the free end of the simply supported compressed member calculated by hand and computed by the ConSteel software using both the **csBeam7** (see **Figure 27**) and the **csShell3** (see **Figure 28**) models. The results are accurate.

Tab.20 Stress analysis of compressed member

section	property	theory [1]	ConSteel			
			csBeam7 [2]	1/2	csShell3 [3]	1/3
HEA300 L=4000mm	e_x [mm]	1,693	1,684	**1,005**	1,717	**0,986**

Notes

In order to compare the results the compressive load on the **csShell3** model was modified by the ratio of the cross-sectional areas computed on the plated structural model and given by the profilARBED catalogue.

WE-21 Bended member

Figure 29 shows a plated structural member which is loaded by uniformly distributed load. The vertical displacement of the middle cross-section and the maximum bending moment of the member are calculated by hand and by the ConSteel software using both of the **csBeam7** and the **csSheel3** finite element models.

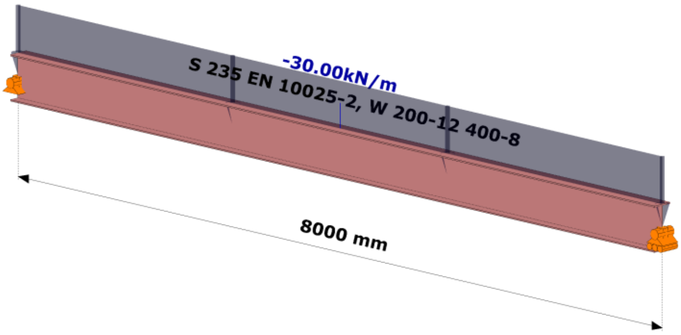


Fig.29 Plated structural member loaded by uniformly distributed load in the vertical plane (welded I section with 200-12 flange and 400-8 web)

A) Calculation by hand

Section: welded symmetric I section

flange $b := 200 \cdot mm$ $\quad t_f := 12 \cdot mm$

web $h_w := 400 \cdot mm$ $\quad t_w := 8 \cdot mm$

Elastic modulus $E := 210000 \cdot \frac{N}{mm^2}$

Length of member $L := 8000 \cdot mm$

Load $p := 30 \cdot \frac{kN}{m}$

Inertia moment $I_y := 2 \cdot b \cdot t_f \cdot \left(\frac{h_w}{2} + \frac{t_f}{2}\right)^2 + t_w \cdot \frac{h_w^3}{12} = 246359467 \cdot mm^4$

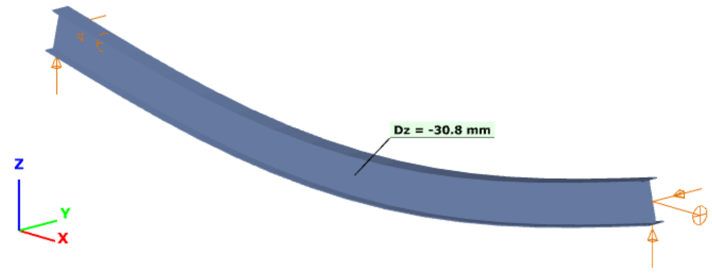
Maximum deflection $e_{z.max} := \frac{5}{384} \cdot \frac{p \cdot L^4}{E \cdot I_y} = 30.927 \cdot mm$

Maximum bending moment $M_{y.max} := \frac{p \cdot L^2}{8} = 240 \cdot kN \cdot m$

B) Computation by ConSteel

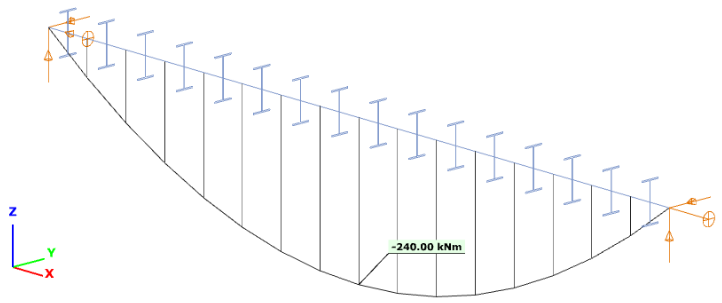
- Beam-column FE model (csBeam7)

  Figure 30 shows the deflections of the member with the numerical value of the maximum deflection. **Figure 31** shows the bending diagram with the maximum bending moment at the middle cross-section (self weight is neglected).



Node	X [mm]	Y [mm]	Z [mm]	Rx [degree]	Ry [degree]	Rz [degree]
10	0,000	0,000	-30,823	0,000	0,000	0,000

Fig.30 *Deflections of the bended member (with n=16 FE)*



Node	Element	N [kN]	$V_{y\,i}$ [kN]	$V_{z\,i}$ [kN]	$M_{x\,i}$ [kNm]	$M_{y\,i}$ [kNm]	$M_{z\,i}$ [kNm]	B [kNm²]
10	8	0,000	0,000	0,000	0,000	-240,000	0,000	0,000

Fig.31 *Bending moment diagram of the bended member*

- Shell FE model (csShell3)

  Figure 32 shows the deflections of the member with the numerical value of the maximum deflection (self weight is neglected).

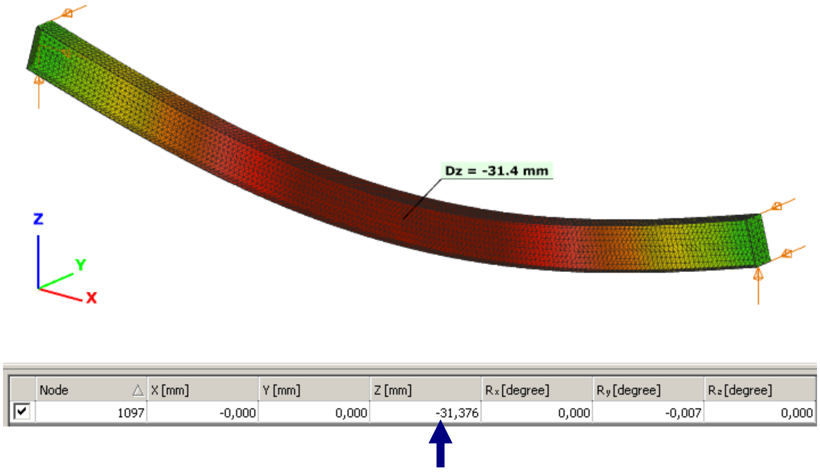


Fig.32 *Deflections of the bended member (with δ=50mm FE size)*

Evaluation

Table 21 shows the maximum value of the vertical deflections calculated by hand and computed by the ConSteel software using both the **csBeam7** and the **csShell3** models. The results are accurate.

Tab.21 Stress analysis of bended member

section	property	theory [1]	ConSteel					
			csBeam7 [2]			csShell3 [3]		
			n	result	1/2	δ	result	1/3
Welded I 200-10 ; 400-8	$e_{z.max}$ [mm]	30.927	4	29,373	1,053	100	31,200	0,991
			6*	30,232	1,023	50	31,376	0,986
			8	30,533	1,013	25	31,427	0,984
			16	30,823	1,003			
	$M_{y.max}$ [kNm]	240	4	240	1,000			
			6*	240				
			8	240				
			16	240				

*) given by the automatic mesh generation (default)

Notes

In the table *n* denotes the number of the finite element in the **csBeam7** model, *δ* denotes the size of the finite elements in [mm] in the **csShell3** model.

The distributed load on the **csBeam7** model is concentrated into the FE nodes, therefore the deflections depend on the number of the finite elements.

The **csShell3** model involves the effect of the shear deformation, therefore it leads greater deflections.

WE-22 Member in torsion (concentrated twist moment)

Figure 33 shows a simple fork supported structural member which is loaded by a concentrated twist moment at the middle cross-section. The member was analysed by hand and by the ConSteel software using both of the **csBeam7** and the **csSheel3** finite element models.

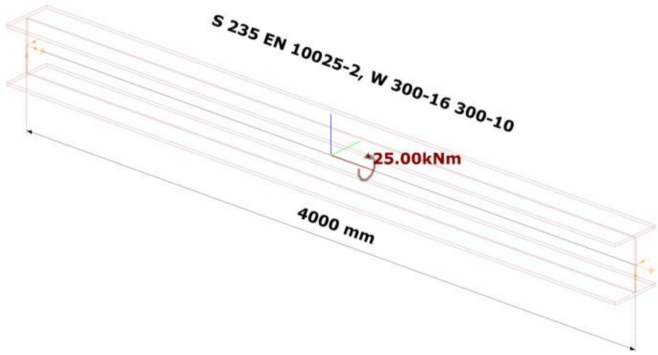


Fig.33 *Simple fork supported structural member loaded by concentrated twist moment at the middle cross-section*

A) Calculation by hand

Section: Welded symmetric I section

flange $b := 300 \cdot mm$ $t_f := 16 \cdot mm$

web $h_w := 300 \cdot mm$ $t_w := 10 \cdot mm$

Sectional properties (by GSS model) $I_t := \frac{1}{3} \cdot \left(2 \cdot b \cdot t_f^{\,3} + h_w \cdot t_w^{\,3}\right) = 919200 \cdot mm^4$

$h_s := h_w + t_f = 316 \cdot mm$

$I_z := 2 \cdot t_f \cdot \frac{b^3}{12} = 72000000\ mm^4$

$I_\omega := I_z \cdot \frac{h_s^{\,2}}{4} = 1797408000000\, mm^6$

$h := h_w + 2 \cdot t_f = 332 \cdot mm$

Elastic modulus $E := 210000 \cdot \frac{N}{mm^2}$ $G := \frac{E}{2 \cdot (1 + 0.3)} = 80769 \cdot \frac{N}{mm^2}$

Parameter $\alpha := \sqrt{\frac{G \cdot I_t}{E \cdot I_\omega}} = 0.444 \frac{1}{m}$

Concentrated torsional moment $M_x := 25 \cdot kN \cdot m$

Member length $L := 4000 \cdot mm$

Cross-section position $L_2 := \frac{L}{2} = 2000 \cdot mm$

Parameters $z := \frac{L}{2} = 2000 \cdot mm$

$z_0 := 0 \cdot mm$

Rotation* $\varphi_{max} := \frac{M_x}{\alpha^2 \cdot E \cdot I_\omega} \cdot \left(\frac{L_2}{L} \cdot z - \frac{\sinh(\alpha \cdot L_2)}{\alpha \cdot \sinh(\alpha \cdot L)} \cdot \sinh(\alpha \cdot z)\right) = 0.067 \cdot rad$

$\varphi_{max.deg} := \varphi_{max} = 3.852 \cdot deg$

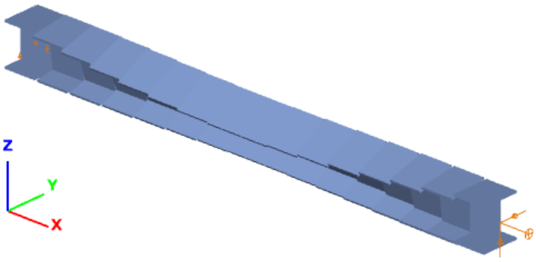
Bimoment*	$B := M_x \cdot \frac{\sinh(\alpha \cdot L_2)}{\alpha \cdot \sinh(\alpha \cdot L)} \cdot \sinh(\alpha \cdot z) = 20.009 kN \cdot m^2$
Torsinal moment*	$M_t := M_x \cdot \left(\frac{L_2}{L} - \frac{\sinh(\alpha \cdot L_2)}{\sinh(\alpha \cdot L)} \cdot \cosh(\alpha \cdot z_0) \right) = 3.696 kN \cdot m$
	$M_\omega := M_x \cdot \frac{\sinh(\alpha \cdot L_2)}{\sinh(\alpha \cdot L)} \cdot \cosh(\alpha \cdot z_0) = 8.804 kN \cdot m$
Check equilibrium	$M_{x.int} := M_t + M_\omega = 12.5 kN \cdot m$
Warping stress	$e_f := \frac{h}{2} - \frac{t_f}{2} = 158 \cdot mm$
	$\omega_{max} := e_f \cdot \frac{b}{2} = 23700 \cdot mm^2$
	$\sigma_{x.max} := \frac{B}{I_\omega} \cdot \omega_{max} = 263.8 \cdot \frac{N}{mm^2}$

*) Csellár, Halász, Réti: Thin-walled steel structures, Muszaki Könvkiadó 1965, Budapest, Hungary, pp. 129-131 (in hungarian)

B) Computation by ConSteel

- Beam-column FE model (csBeam7)

Figure 34 shows the deflections of the member with the numerical value of the maximum rotation (self weight is neglected). **Figure 35** shows the bimoment diagram with the maximum bimoment at the middle cross-section. **Figure 36** shows the warping normal stress in the middle cross-section.



Node	X [mm]	Y [mm]	Z [mm]	Rx [degree]
3	0,000	0,000	0,000	3,854

Fig.34 *Rotation of the member due to concentrated twist moment*

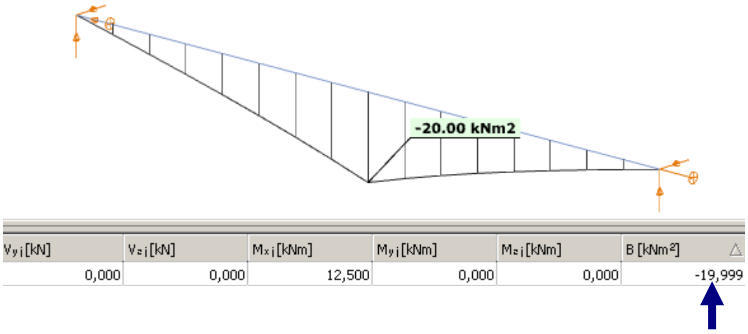


Fig.35 *Bimoment of the member due to concentrated twist moment*

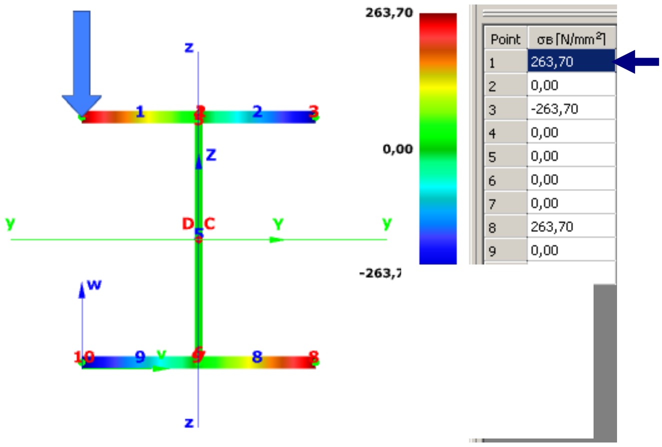


Fig.36 *Warping normal stress in the middle cross-section*

- Shell FE model (csShell3)

  Figure 37 shows the rotation of the member with the numerical value of the maximum rotation (self weight is neglected). **Figure 38** shows the axial stress distribution in the middle cross-section.

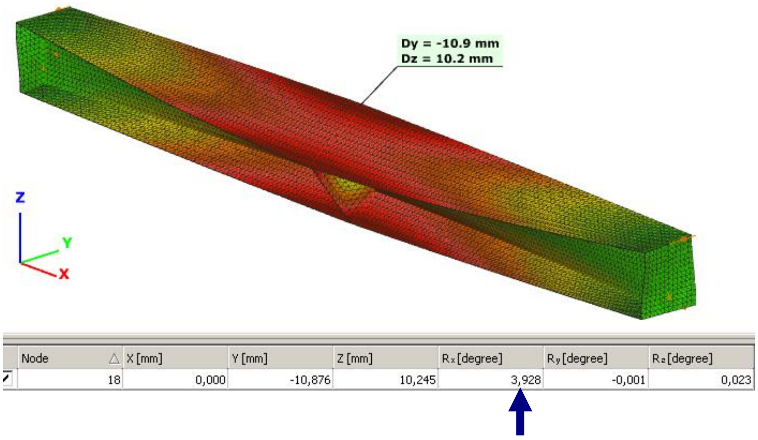


Fig.37 *Maximum rotation of the middle cross-section*

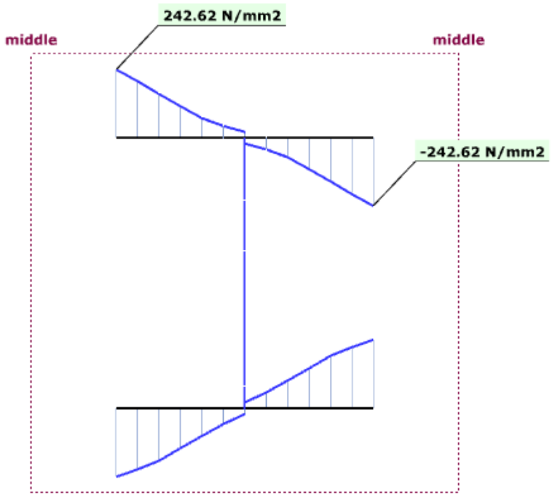


Fig.38 Axial stress distribution in the middle cross-section (with 25mm FE)

Evaluation

Table 22 shows the value of the rotation, bimoment and the axial stress of the middle cross-section calculated by hand and computed by the ConSteel software using both the **csBeam7** and the **csShell3** models. The results are accurate.

Tab.22 Stress analysis of bended member

section	property	theory [1]	ConSteel					
			csBeam7 [2]			csShell3 [3]		
			n	result	1/2	δ	result	1/3
Welded I f: 300-16 w: 300-10	$R_{x.max}$ [deg]	3,852	2	3,852	**1,000**	50	4,021	**0,958**
			4	3,854	**0,999**	25	3,928	**0,981**
			6*	3,854	**0,999**	12,5	3,922	**0,982**
			16	3,854	**0,999**			
	B_{max} [kNm^2]	20,00	2	20,00	**1,000**			
			4	20,00	**1,000**			
			6*	20,00	**1,000**			
			16	19,99	**1,001**			
	$\sigma_{\omega.max}$** [N/mm^2]	263,8	2	263,7	**1,000**	50	213,8	**1,234**
			4	263,7	**1,000**	25	242,6	**1,061**
			6*	263,7	**1,000**	12,5	261,4	**1,009**
			16	263,7	**1,000**			

*) given by automatic mesh generation (default)

**) in middle line of the flange

Notes

In the table *n* denotes the number of finite element in the **csBeam7** model, *δ* denotes the size of the finite elements in [mm] in the **csShell3** model.

WE-23 Member in torsion (torsion by transverse concentrated load on mono-symmetric I section)

Figure 39 shows a simple fork supported member with mono-symmetric welded I section which is loaded by a concentrated transverse force in the centroid of the middle cross-section. The member was analysed by hand and by the ConSteel software using both of the **csBeam7** and the **csSheel3** finite element models.

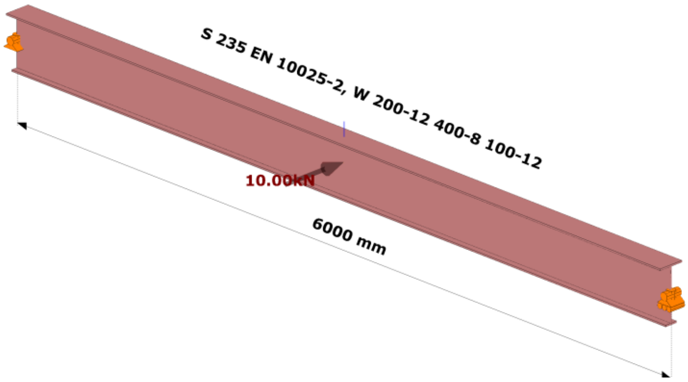


Fig.39 Simple fork supported member with mono-symmetric welded I section loaded by concentrated transverse force in the centroid

A) Calculation by hand

Section: Welded monsymmetric I section

top flange	$b_1 := 200 \cdot mm$	$t_{f1} := 12 \cdot mm$
web	$h_w := 400 \cdot mm$	$t_w := 8 \cdot mm$
bottom flange	$b_2 := 100 \cdot mm$	$t_{f2} := 12 \cdot mm$

Sectional properties

$$I_{z1} := t_{f1} \cdot \frac{b_1^{\,3}}{12} = 8000000 \cdot mm^4 \qquad I_{z2} := t_{f2} \cdot \frac{b_2^{\,3}}{12} = 1000000 \cdot mm^4$$

$$I_z := I_{z1} + I_{z2} = 9000000 \cdot mm^4$$

$$I_t := \frac{1}{3} \cdot \left(b_1 \cdot t_{f1}^{\,3} + b_2 \cdot t_{f2}^{\,3} + h_w \cdot t_w^{\,3} \right) = 241067 \cdot mm^4$$

$$\beta_f := \frac{I_{z1}}{I_{z1} + I_{z2}} = 0.889 \qquad h_s := h_w + \frac{t_{f1}}{2} + \frac{t_{f2}}{2} = 412 \cdot mm$$

$$I_\omega := \beta_f \cdot (1 - \beta_f) \cdot I_z \cdot h_s^{\,2} = 1.5088 \times 10^{11} \cdot mm^6$$

$z_S := 248.4 \cdot mm$ (by GSS model of ConSteel)

$z_D := 123.4 \cdot mm$ (by GSS model of ConSteel)

Elastic modulus $E := 210000 \cdot \frac{N}{mm^2}$ $\quad G := \frac{E}{2 \cdot (1 + 0.3)} = 80769 \cdot \frac{N}{mm^2}$

Parameter $\alpha := \sqrt{\frac{G \cdot I_t}{E \cdot I_\omega}} = 0.784 \frac{1}{m}$

Member length $L := 6000 \cdot mm$

Transverse force $F_y := 10 \cdot kN$

Torsional moment $M_x := F_y \cdot z_D = 1.234 kN \cdot m$

Cross-section position $L_2 := \frac{L}{2} = 3000 \cdot mm$

$z := \frac{L}{2} = 3000 \cdot mm \qquad z_0 := 0 \cdot mm$

Rotation* $\varphi_{max} := \frac{M_x}{\alpha^2 \cdot E \cdot I_\omega} \cdot \left(\frac{L_2}{L} \cdot z - \frac{\sinh(\alpha \cdot L_2)}{\alpha \cdot \sinh(\alpha \cdot L)} \cdot \sinh(\alpha \cdot z) \right) = 3.172 \cdot deg$

Bimoment* $B := M_x \cdot \frac{\sinh(\alpha \cdot L_2)}{\alpha \cdot \sinh(\alpha \cdot L)} \cdot \sinh(\alpha \cdot z) = 0.773 kN \cdot m^2$

Torsinal moment* $M_t := M_x \cdot \left(\frac{L_2}{L} - \frac{\sinh(\alpha \cdot L_2)}{\sinh(\alpha \cdot L)} \cdot \cosh(\alpha \cdot z_0) \right) = 0.501 kN \cdot m$

$M_\omega := M_x \frac{\sinh(\alpha \cdot L_2)}{\sinh(\alpha \cdot L)} \cdot \cosh(\alpha \cdot z_0) = 0.116 kN \cdot m$

Check equilibrium $M_{x.int} := M_t + M_\omega = 0.617 kN \cdot m$

Warping stress $\omega_2 := 18311 \cdot mm^2$ (by GSS model of ConSteel)

$\sigma_{\omega.2} := \frac{B}{I_\omega} \cdot \omega_2 = 93.8 \cdot \frac{N}{mm^2}$

Bending moment $M_z := F_y \cdot \frac{L}{4} = 15 kN \cdot m$

Bending stress $\sigma_{Mz2} := \frac{M_z}{I_z} \cdot \frac{b_2}{2} = 83.33 \cdot \frac{N}{mm^2}$

Axial stress in bottom flange $\sigma_{x2} := \sigma_{\omega.2} + \sigma_{Mz2} = 177.14 \cdot \frac{N}{mm^2}$

*) Csellár, Halász, Réti: Thin-walled steel structures, Muszaki Könvkiadó 1965, Budapest, Hungary, pp. 129-131 (in Hungarian)

B) Computation by ConSteel

- Beam-column FE model (csBeam7)

Figure 40 shows the deformated member with the numerical value of the maximum rotation (self weight is neglected). **Figure 41** shows the bimoment diagram with the maximum bimoment at the middle cross-section. **Figure 42** shows the warping normal stress in the middle cross-section.

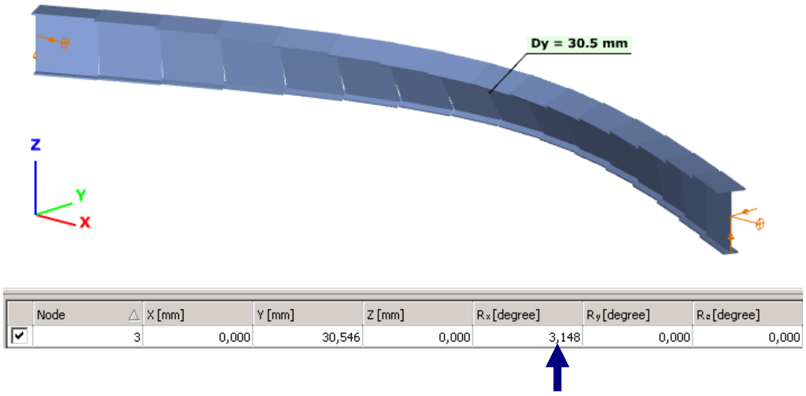


Fig.40 *Rotation of the member due to concentrated transverse force in the centroid of the middle cross-section (n=16)*

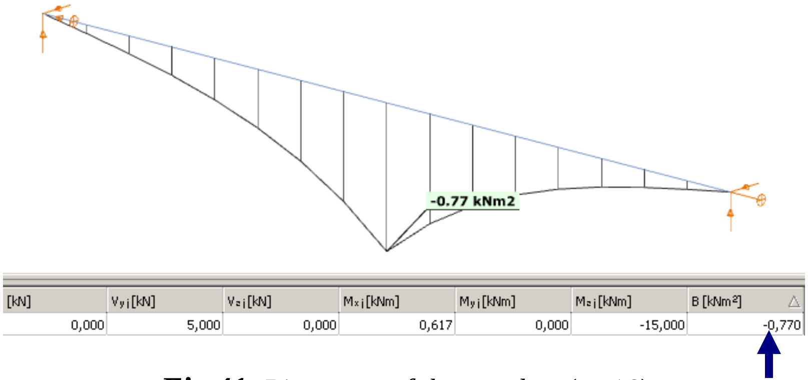


Fig.41 *Bimoment of the member (n=16)*

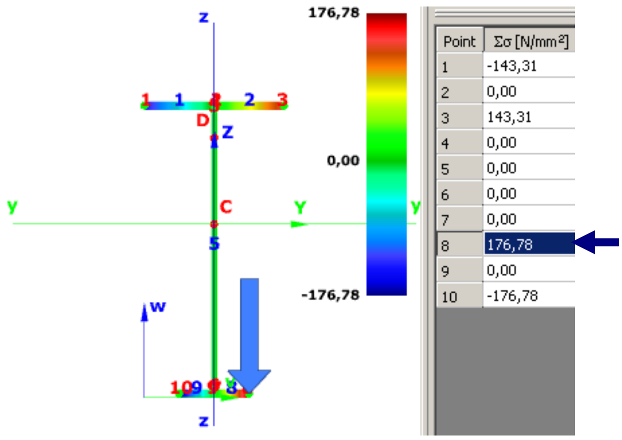


Fig.42 *Warping normal stress in the middle cross-section (n=16)*

Evaluation

Table 23 shows the value of the rotation, bimoment and the axial stress of the middle cross-section calculated by hand and computed by the ConSteel software using both the **csBeam7** and the **csShell3** models. The results are accurate.

Tab.23 Stress analysis of member in torsion

section	property	theory [1]	ConSteel					
			csBeam7 [2]			csShell3 [3]		
			n	result	1/2	δ	result	1/3
Welded I 200-12 400-8 100-12	$R_{x.max}$ [deg]	3,172	2	3,122	1,016	50	2,996	1,059
			4	3,145	1,009	25	3,133	1,013
			8*	3,148	1,007	12,5	3,173	1,000
			16	3,148	1,007			
	B_{max} [kNm^2]	0,773	2	0,779	0,992			
			4	0,771	1,003			
			8*	0,770	1,004			
			16	0,770	1,004			
	$\sigma_{.max}$** [N/mm^2]	177,1	2	177,9	0,996	50	165,3	1,072
			4	176,9	1,001	25	173,4	1,021
			8*	176,8	1,001	12,5	176,1	1,006
			16	176,8	1,001			

*) given by the automatic mesh generation (default)
**) in the middle plane of the flange

Notes

In the Table 23 *n* denotes the number of the finite elements of the **csBeam7** model, *δ* denotes the size of the shell finite elements in [mm] in the **csShell3** model.

2.2.2 Geometrically nonlinear (second order) theory

The geometrically nonlinear analysis of simple structural members using the ConSteel software (based on the **csBeam7** and the **csShell3** finite element) are checked in the following **Worked Examples** (WE-24 to WE-25).

WE-24 Member subjected to bending and compression

Figure 43 shows a simple fork supported member with IPE360 section subjected to axial force and bending about the minor axis due to lateral distributed force. The deflection and the maximum compressive stress of the member are calculated by hand and by the ConSteel software using the **csBeam7** model.

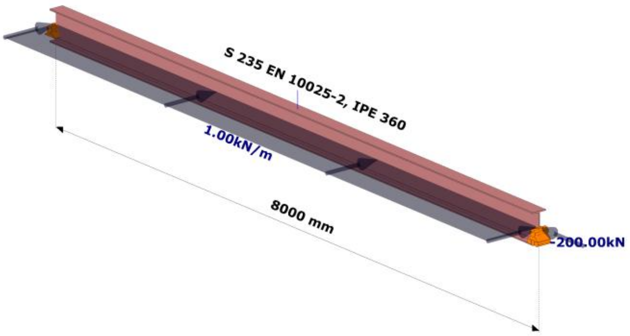


Fig.43 *Simple fork supported member with IPE360 section subjected to transverse load and compressive force*

A) Calculation by hand

Section: **IPE 360**

Sectional properties (ProfilARBED) $A := 7273 \cdot mm^2$

$I_z := 10430000\ mm^4$

Elastic modulus $E = 210000 \cdot \dfrac{N}{mm^2}$

Length of member $L := 8000 \cdot mm$

Distributed load intensity $p := 1 \cdot \dfrac{kN}{m}$

Compressive force $F_x := 200 \cdot kN$

Crirical foce $F_{cr.x} := \dfrac{\pi^2 \cdot E \cdot I_z}{L^2} = 337.8 \cdot kN$

Bending moment by first order theory $M_{z1} := \dfrac{p \cdot L^2}{8} = 8\,kN \cdot m$

Moment amplifier factor $\eta := \dfrac{1}{1 - \dfrac{F_x}{F_{cr.x}}} = 2.452$

Bending moment by second order theory $M_{z2} := \eta \cdot M_{z1} = 19.61\,kN \cdot m$

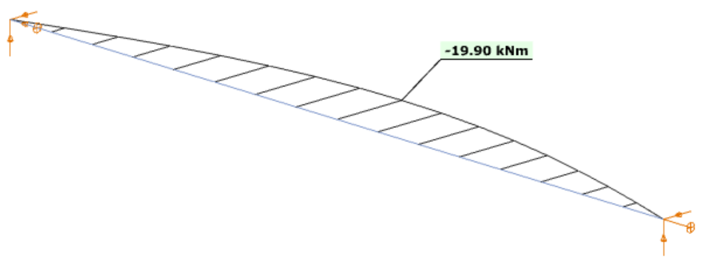
Maximum compressive stress $y_{max} := 85 \cdot mm$

$$\sigma_{c.max} := \frac{F_x}{A} + \frac{M_{z2}}{I_z} \cdot y_{max} = 187.3 \cdot \frac{N}{mm^2}$$

B) Computation by ConSteel

- Beam-column FE model (csBeam7)

  Figure 44 shows the second order bending moment diagram of the member which was computed by the ConSteel software using the **csBeam7** finite element model.



N [kN]	$V_{y,i}$ [kN]	$V_{z,i}$ [kN]	$M_{x,i}$ [kNm]	$M_{y,i}$ [kNm]	$M_{z,i}$ [kNm]	B [kNm²]
-200,000	0,000	0,000	0,000	0,000	-19,900	0,000

Fig.44 *Bending moment diagram of the member (n=16)*

Evaluation

Table 24 shows the second order bending moment and the maximum axial compressive stress value of the middle cross-section calculated by hand and computed by the ConSteel software using the **csBeam7** model. The results are accurate.

Tab.24 Second order stress analysis of member in bending and compression

section	property	theory [1]	csBeam7 [2]		
			n	result	1/2
IPE360	$M_{z.max}$ [kNm]	19,61	2	17,40	1,127
			4	19,33	1,015
			6*	19,67	0,997
			16	19,90	0,985
	$\sigma_{c.max}$ [N/mm^2]	187,3	2	169,7	1,104
			4	185,5	1,010
			6*	188,3	0,995
			16	190,2	0,985

*) given by the automatic mesh generation (default)

Notes

In the Table 23 *n* denotes the number of the finite elements of the **csBeam7** model.

WE-25 Member subjected to biaxial bending and compression

Figure 45 shows a simple fork supported member with IPE360 equivalent welded section (flange: 170-12,7; web: 347-8) subjected to biaxial bending about the minor axis due to concentrated end moments and to compressive force. Deflections of middle cross-section of the member are calculated by hand and by the ConSteel software using both of **csBeam7** model and **csShell3** model.

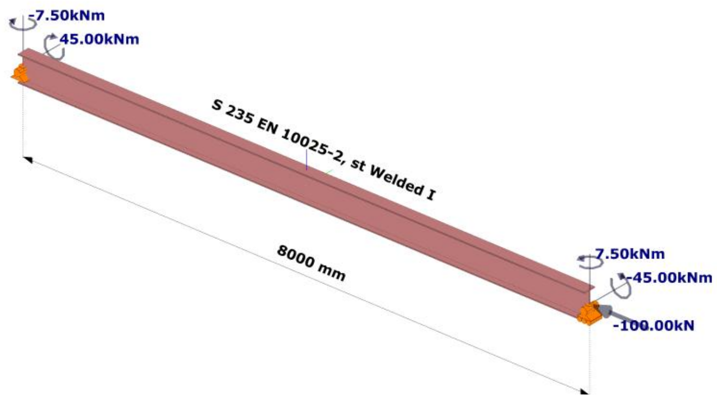


Fig.45 *Simple fork supported member with IPE360 section subjected to biaxial bending and compression*

A) Calculation by hand (using approximated method)

Section: **IPE360 equivalent welded I section**

Sectional properties (by EPS model) $A := 6995 \cdot mm^2$

$I_y := 155238000\ mm^4$ $\quad I_z := 10413000\ mm^4$

$I_t := 291855 \cdot mm^4$ $\quad I_\omega := 313000000000\ mm^6$

$$r_0 := \sqrt{\frac{I_y}{A} + \frac{I_z}{A}} = 153.887\,mm$$

Elastic modulus $E = 210000 \frac{N}{mm^2}$ $\quad G := \frac{E}{2 \cdot (1 + 0.3)} = 80769 \frac{N}{mm^2}$

Length of member $L := 8000 \cdot mm$

Compressive force $P := 100 \cdot kN$

End moments $M_y := 45 \cdot kN \cdot m$ $\quad M_z := 7.5 \cdot kN \cdot m$

Critical axial forces

$$P_{cr.y} := \frac{\pi^2 \cdot E \cdot I_y}{L^2} = 5027 \cdot kN$$

$$P_{cr.z} := \frac{\pi^2 \cdot E \cdot I_z}{L^2} = 337.2 \cdot kN$$

$$P_{cr.\omega} := \frac{1}{r_0^2} \cdot \left(\frac{\pi^2 \cdot E \cdot I_\omega}{L^2} + G \cdot I_t \right) = 1423.5 \cdot kN$$

Displacements*

$$C := \frac{\pi^2}{8} \cdot \frac{M_y \cdot M_z}{P_{cr.y} \cdot P_{cr.z}} \cdot P \cdot \frac{\frac{P_{cr.y}}{P_{cr.z} - P} - \frac{P_{cr.z}}{P_{cr.y} - P} - \frac{4}{\pi} \cdot \frac{P_{cr.z} - P_{cr.y}}{P}}{\frac{M_y^2}{P_{cr.z} - P} + \frac{M_z^2}{P_{cr.y} - P} - r_0^2 \cdot \left(P_{cr.\omega} - P\right)} = -0.087$$

$$u_{max} := -\frac{1}{P_{cr.z} - P} \cdot \left(\frac{\pi^2}{8} \cdot M_z - C \cdot M_y \right) = -55.53\,mm$$

$$v_{max} := \frac{1}{P_{cr.y} - P} \cdot \left(\frac{\pi^2}{8} \cdot M_y + C \cdot M_x \right) = 11.25\,mm$$

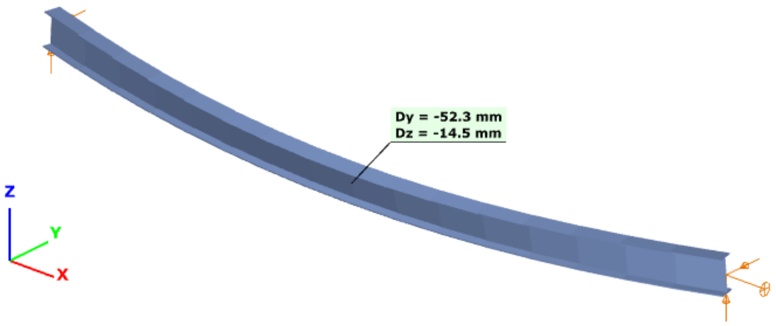
$$\varphi_{max} := C = -4.991\,deg$$

*) Chen, W. and Atsuta, T.: Theory of Beam-Columns, Vol. 2: Space behavior and design, McGRAW-HILL 1977, p. 192

B) Computation by ConSteel

- Beam-column FE model (csBeam7)

  Figure 46 shows the second order deflection of the member which was computed by the ConSteel software using the **csBeam7** finite element model.

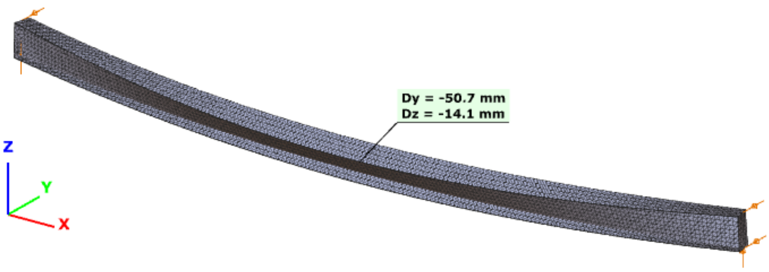


Node	X [mm]	Y [mm]	Z [mm]	R_x [degree]	R_y [degree]	R_z [degree]
10	0,000	-52,312	-14,506	5,068	0,000	0,000

Fig.46 *Deformation of the member by csBeam7 FE model (n=16)*

- Shell FE model (csShell3)

  Figure 47 shows the second order deflection of the member which was computed by the ConSteel software using the **csShell3** finite element model.



Node	X [mm]	Y [mm]	Z [mm]	R_x [degree]	R_y [degree]	R_z [degree]
1821	-0,000	-50,732	-14,139	5,185	-0,004	0,018

Fig.47 *Deformation of the member by csShell3 FE model (δ=43mm)*

Evaluation

Table 25 shows the second order bending moment and the maximum axial compressive stress value of the middle cross-section calculated by approximated theory and computed by the ConSteel software using the **csBeam7** and **csShell3** model. The accuracy of the approximated hand calculation is a bit pure, but the ConSteel results of **csBeam7** model comparing with the **csShell3** model are accurate.

Tab.25 Second order stress analysis of member in bending and compression

section	displacement	theory (approximation)	ConSteel			
			csBeam7		**csShell3**	
			n	**result**	**δ**	**result**
IPE360 equivalent welded I section 170-12,7 347-8	$e_{y.max}$ [mm]	55,53	2	53,00	43	51,17
			4	53,38	25	53,03
			6*	53,46	$\frac{csBeam(n=16)}{csShell(\delta=25)} = 1{,}009$	
			16	53,50		
	$e_{z.max}$ [mm]	11,25	2	11,10	43	10,81
			4	11,10	25	10,83
			6*	11,10	$\frac{csBeam(n=16)}{csShell(\delta=25)} = 1{,}025$	
			16	11,10		
	$\varphi_{.max}$[deg]	4,991	2	4,172	43	4,287
			4	4,216	25	4,433
			6*	4,229	$\frac{csBeam(n=16)}{csShell(\delta=25)} = 0{,}956$	
			16	4,239		

*) given by the automatic mesh generation (default)

Notes

In the **Table 25** **n** denotes the number of the finite elements of the **csBeam7** model, **δ** denotes the maximum size of the shell finite elements of the **csShell3** model in [mm].

2.3 Stability analysis

The stability analysis of simple structural members using the ConSteel software based on both of the **csBeam7** and optionally the **csShell3** finite element models are checked in the following **Worked Examples** (**WE-26** to **WE-33**).

WE-26 Lateral torsional buckling (double symmetric section & constant bending moment)

Figure 48 shows a simple fork supported member with welded section (flange: 200-12; web: 400-8) subjected to bending about the major axis due to concentrated end moments. Critical moment of the member is calculated by hand and by the ConSteel software using the **csBeam7** model.

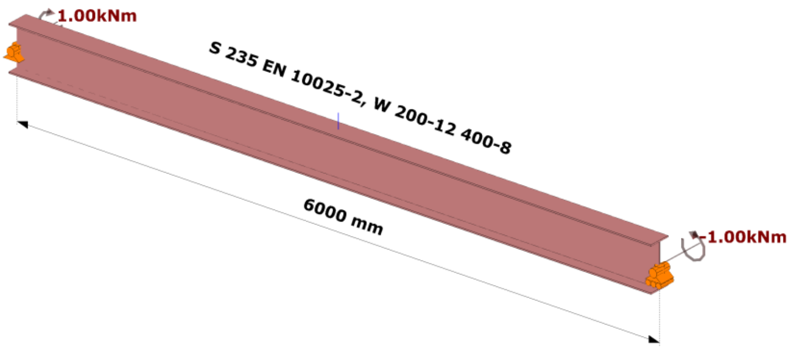


Fig.48 *Simple fork supported member subjected to bending about the major axis (LTB)*

A) Calculation by hand

Section: welded symmetric I section

flange $b := 200 \cdot mm$ $\quad t_f := 12 \cdot mm$

web $h_w := 400 \cdot mm$ $\quad t_w := 8 \cdot mm$

Sectional properties

$$I_z := 2 \cdot t_f \cdot \frac{b^3}{12} = 16000000\ mm^4$$

$$I_t := \frac{1}{3} \cdot \left(2 \cdot b \cdot t_f^{\,3} + h_w \cdot t_w^{\,3}\right) = 298667 \cdot mm^4$$

$$I_\omega := \frac{t_f \cdot b^3}{24} \cdot \left(h_w + t_f\right)^2 = 678976000000\ mm^6$$

Elastic modulus

$$E := 210000 \cdot \frac{N}{mm^2} \qquad G := \frac{E}{2 \cdot (1 + 0.3)} = 80769 \cdot \frac{N}{mm^2}$$

Member length $L := 6000 \cdot mm$

Critical moment

$$M_{cr} := \frac{\pi^2 \cdot E \cdot I_z}{L^2} \cdot \sqrt{\frac{I_\omega}{I_z} + \frac{L^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z}} = 241.31 kN \cdot m$$

B) Computation by ConSteel

- Beam-column FE model (csBeam7)

  Figure 49 shows the member subjected to lateral torsional buckling which was computed by the ConSteel software using the **csBeam7** finite element model.

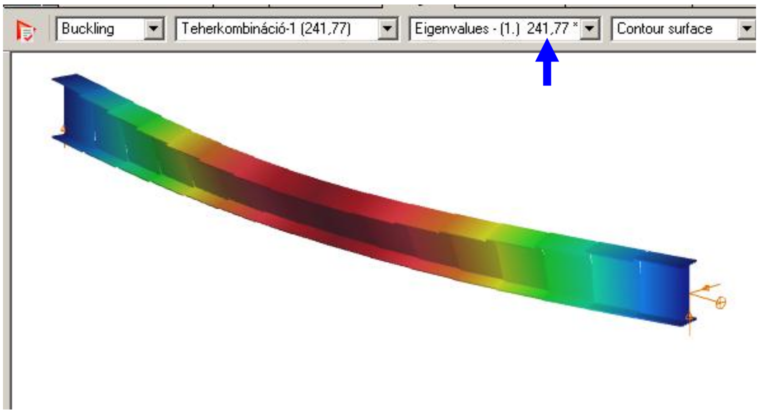


Fig.49 *LTB of simple supported structural member (n=16)*

Evaluation

Table 26 shows the critical moment for lateral torsional buckling of the member which calculated by hand and computed by the ConSteel software using the **csBeam7** model. The result is accurate.

Tab.57 Stability analysis of member on compression (L=4000mm)

section	**critical force**	**theory** [1]	**csBeam7** [2]		
			n	**result**	**1/2**
Welded I 200-12 ; 400-8	M_{cr} [kNm]	241,31	2	243,24	**0,992**
			4	241,87	**0,998**
			6*	241,79	**0,998**
			16	241,77	**0,998**

*) given by the automatic mesh generation (default)

Note

In the **Table 57** **n** denotes the number of the finite elements of the **csBeam7** model.

WE-27 Lateral torsional buckling (double symmetric section & triangular bending moment distribution)

Figure 50 shows a simple fork supported member with welded section (flange: 200-12; web: 400-8) subjected to transverse force at middle cross section in the main plane of the member. The critical force is calculated by hand and by the ConSteel software using **csBeam7** model.

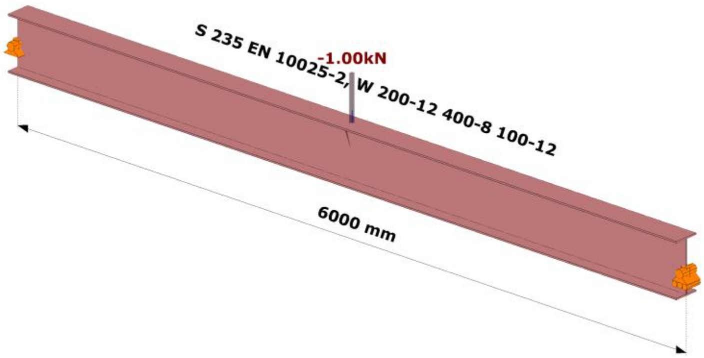


Fig.50 *Simple fork supported member subjected to transverse force (LTB)*

A) Calculation by hand

Section: welded symmetric I section

flange $b := 200 \cdot mm$ $t_f := 12 \cdot mm$

web $h_w := 400 \cdot mm$ $t_w := 8 \cdot mm$

Sectional properties $I_z := 2 \cdot t_f \cdot \frac{b^3}{12} = 16000000\ mm^4$

$$I_t := \frac{1}{3} \cdot \left(2 \cdot b \cdot t_f^3 + h_w \cdot t_w^3\right) = 298667 \cdot mm^4$$

$$I_\omega := \frac{t_f \cdot b^3}{24} \cdot \left(h_w + t_f\right)^2 = 678976000000\ mm^6$$

Elastic modulus $E := 210000 \cdot \frac{N}{mm^2}$ $G := \frac{E}{2 \cdot (1 + 0.3)} = 80769 \cdot \frac{N}{mm^2}$

Member length $L := 6000 \cdot mm$

Critical force $C_1 := 1.365$

$$M_{cr} := C_1 \frac{\pi^2 \cdot E \cdot I_z}{L^2} \cdot \sqrt{\frac{I_\omega}{I_z} + \frac{L^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z}} = 329.387 kN \cdot m$$

$$F_{cr} := 4 \cdot \frac{M_{cr}}{L} = 219.6 \cdot kN$$

B) Computation by ConSteel

- Beam-column FE model (csBeam7)

  Figure 51 shows the LTB of the member subjected to transverse force. The critical force is computed by the ConSteel software using **csBeam7** finite element model.

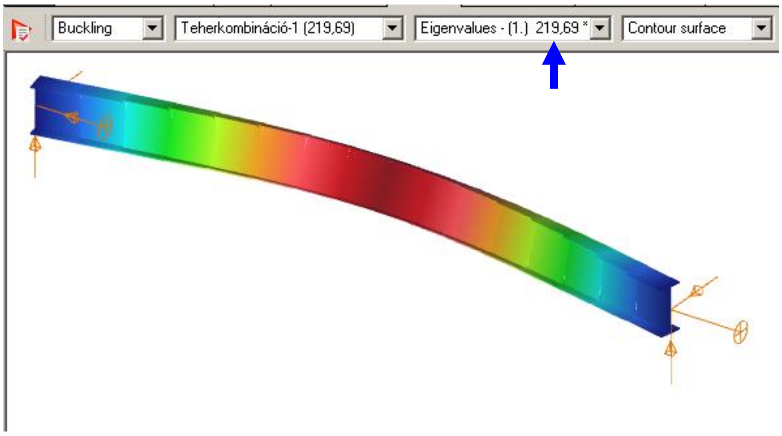


Fig.51 *LTB of simple supported structural member subjected to transverse force (n=16)*

Evaluation

Table 27 shows the critical force for lateral torsional buckling of the member which calculated by hand and computed by the ConSteel software using **csBeam7** model. The result is accurate.

Tab.27 Stability analysis of member on compression (L=4000mm)

section	critical force	theory [1]	csBeam7 [2]		
			n	**result**	**1/2**
Welded I 200-12 ; 400-8	P_{cr} [kN]	219,6	2	220,9	**0,994**
			4	219,9	**0,999**
			6*	219,7	**1,000**
			16	219,7	**1,000**

*) given by the automatic mesh generation (default)

Note

In the **Table 27** **n** denotes the number of the finite elements of the **csBeam7** model.

WE-28 Lateral torsional buckling (mono-symmetric section & constant moment)

Figure 52 shows a simple fork supported member with welded mono-symmetric I section (flange: 200-12 and 100-12; web: 400-8) subjected to equal end moments. The critical moment is calculated by hand and by the ConSteel software using **csBeam7** and **csShell3** models.

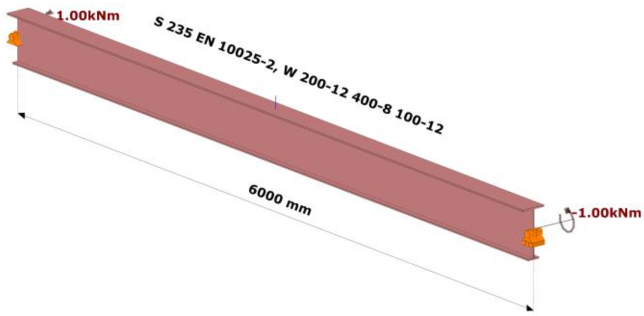


Fig.52 *Simple fork supported member with mono-symmetric I section subjected to equal end moments (LTB)*

A) Calculation by hand

Section: welded mono-symmetric I section

top flange	$b_1 := 200 \cdot mm$	$t_{f1} := 12 \cdot mm$
web	$h_w := 400 \cdot mm$	$t_w := 8 \cdot mm$
bottom flange	$b_2 := 100 \cdot mm$	$t_{f2} := 12 \cdot mm$

Sectional properties

$Z_S := 248.4 \cdot mm$ (by GSS model of ConSteel)

$z_D := 123.4 \cdot mm$ (by GSS model of ConSteel)

$$I_{z1} := t_{f1} \cdot \frac{b_1^3}{12} = 8000000 \cdot mm^4 \qquad I_{z2} := t_{f2} \cdot \frac{b_2^3}{12} = 1000000 \cdot mm^4$$

$$I_z := I_{z1} + I_{z2} = 9000000 \cdot mm^4$$

$I_y := 186493000 \cdot mm^4$ (by GSS model of ConSteel)

$$I_t := \frac{1}{3} \cdot \left(b_1 \cdot t_{f1}^3 + b_2 \cdot t_{f2}^3 + h_w \cdot t_w^3 \right) = 241067 \cdot mm^4$$

$$\beta_f := \frac{I_{z1}}{I_{z1} + I_{z2}} = 0.889$$

$$h_s := h_w + \frac{t_{f1}}{2} + \frac{t_{f2}}{2} = 412 \cdot mm$$

$$I_\omega := \beta_f \cdot \left(1 - \beta_f\right) \cdot I_z \cdot h_s^2 = 150883555556 \, mm^6$$

$$e := h_w + t_{f2} + \frac{t_{f1}}{2} - Z_S = 169.6 \cdot mm$$

$$A_1 := b_1 \cdot t_{f1} = 2400 \cdot mm^2 \qquad A_2 := b_2 \cdot t_{f2} = 1200 \cdot mm^2$$

$$q_x := \frac{1}{I_y} \cdot \left[z_D \cdot I_{z1} + A_1 \cdot e^3 - A_2 \cdot \left(h_s - e\right)^3 + \frac{t_w}{4} \cdot \left[e^4 - \left(h_s - e\right)^4 \right] \right] = -51.725 \cdot mm$$

$$z_j := z_D - 0.5 q_x = 149.262 \cdot mm$$

Elastic modulus $E := 210000 \cdot \frac{N}{mm^2}$ $\qquad G := \frac{E}{2 \cdot (1 + 0.3)} = 80769 \cdot \frac{N}{mm^2}$

Member length $L := 6000 \cdot mm$

Critical moment

$$M_{cr} := \frac{\pi^2 \cdot E \cdot I_z}{L^2} \cdot \left(\sqrt{\frac{I_\omega}{I_z} + \frac{L^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z} + z_j^2} + z_j \right) = 220.77 kN \cdot m$$

B) Computation by ConSteel

- Beam-column FE model (csBeam7)

Figure 53 shows the LTB of the mono-symmetric member subjected to equal end moments. The critical moment is computed by the ConSteel software using **csBeam7** finite element model.

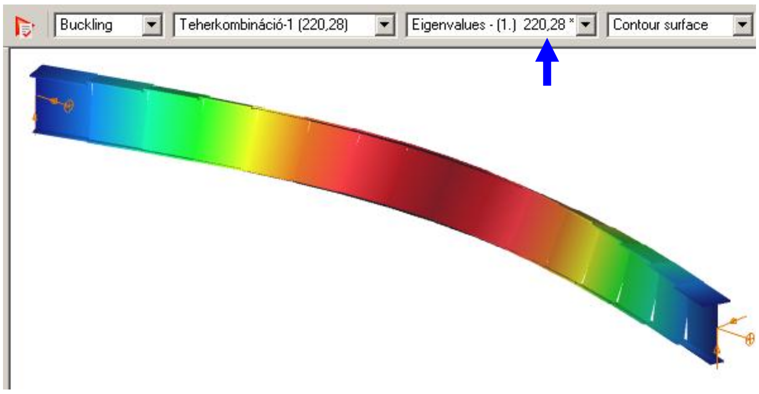


Fig.53 *LTB of simple supported mono-symmetric structural member subjected to equal end moments (n=16)*

- Shell FE model (csShell3)

Figure 54 shows the LTB of the mono-symmetric member subjected to equal end moments. The critical force is computed by the ConSteel software using **csShell3** finite element model.

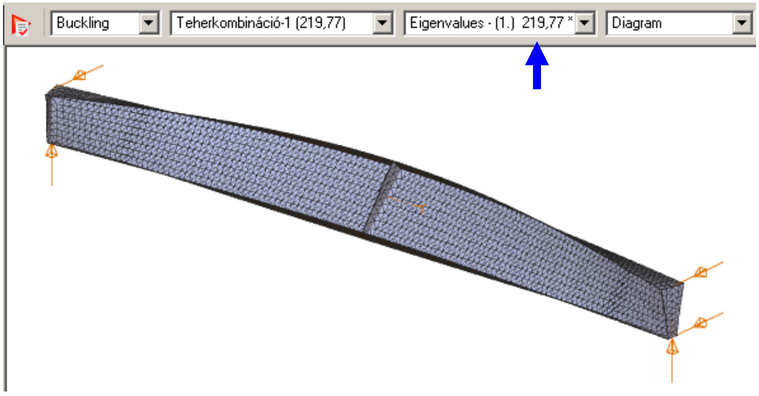


Fig.54 *LTB of simple supported mono-symmetric structural member subjected to equal end moments (δ=50mm)*

Evaluation

Table 28 shows the critical moment for lateral torsional buckling of the member which calculated by hand and computed by the ConSteel software using **csBeam7** and **csShell3** models. The result is accurate.

Tab.28 Stability analysis of mono-symmetric member subjected to equal end moments

section	critical force	theory [1]	csBeam7 [2]			csShell3 [3]		
			n	**result**	**1/2**	**δ**	**result**	**1/3**
Welded mono-symmetric I 200-12 ; 400-8 ; 100-12	M_{cr} [kNm]	220,77	2	221,67	**0,996**	50	219,77	**1,005**
			4	220,37	**1,002**	25	217,13	**1,016**
			6*	220,30	**1,002**			
			16	220,28	**1,002**			

*) given by the automatic mesh generation (default)

Note

In the **Table 28 n** denotes the number of the finite elements of the **csBeam7** model, **δ** denotes the maximum shell FE size.

WE-29 Lateral torsional buckling (mono-symmetric section & triangular moment distribution)

Figure 55 shows a simple fork supported member with welded mono-symmetric I section (flange: 200-12 and 100-12; web: 400-8) subjected to transverse force at the middle cross-section of the member. The critical force is calculated by hand and by the ConSteel software using **csBeam7** and **csShell3** models.

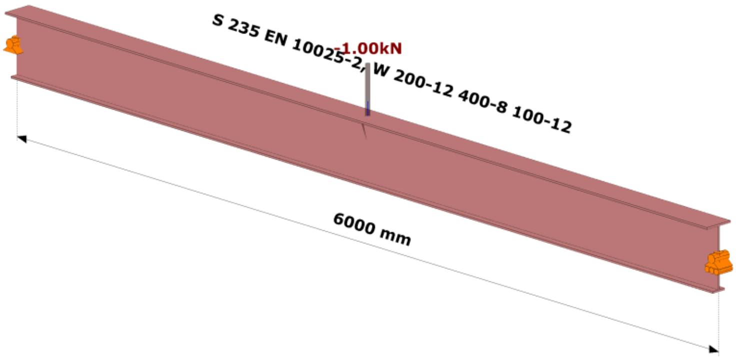


Fig.55 *Simple fork supported member with mono-symmetric welded I section subjected to transverse force (LTB)*

A) Calculation by hand

Section: welded monsymmetric I section

top flange $b_1 := 200 \cdot mm$ $t_{f1} := 12 \cdot mm$

web $h_w := 400 \cdot mm$ $t_w := 8 \cdot mm$

bottom flange $b_2 := 100 \cdot mm$ $t_{f2} := 12 \cdot mm$

Sectional properties $Z_S := 248.4 \cdot mm$ (by GSS model of ConSteel)

$z_D := 123.4 \cdot mm$ (by GSS model of ConSteel)

$$I_{z1} := t_{f1} \cdot \frac{b_1^3}{12} = 8000000 \cdot mm^4 \qquad I_{z2} := t_{f2} \cdot \frac{b_2^3}{12} = 1000000 \cdot mm^4$$

$$I_z := I_{z1} + I_{z2} = 9000000 \cdot mm^4$$

$I_y := 186493000 \cdot mm^4$ (by GSS model of ConSteel)

$$I_t := \frac{1}{3} \cdot \left(b_1 \cdot t_{f1}^3 + b_2 \cdot t_{f2}^3 + h_w \cdot t_w^3\right) = 241067 \cdot mm^4$$

$$\beta_f := \frac{I_{z1}}{I_{z1} + I_{z2}} = 0.889$$

$$h_s := h_w + \frac{t_{f1}}{2} + \frac{t_{f2}}{2} = 412 \cdot mm$$

$$I_\omega := \beta_f \cdot \left(1 - \beta_f\right) \cdot I_z \cdot h_s^2 = 150883555556\, mm^6$$

$$e := h_w + t_{f2} + \frac{t_{f1}}{2} - Z_S = 169.6 \cdot mm$$

$$A_1 := b_1 \cdot t_{f1} = 2400 \cdot mm^2 \qquad A_2 := b_2 \cdot t_{f2} = 1200 \cdot mm^2$$

$$q_x := \frac{1}{I_y} \cdot \left[z_D \cdot I_{z1} + A_1 \cdot e^3 - A_2 \cdot \left(h_s - e\right)^3 + \frac{t_w}{4} \cdot \left[e^4 - \left(h_s - e\right)^4\right]\right] = -51.725 \cdot mm$$

$$z_j := z_D - 0.5 q_x = 149.262 \cdot mm$$

Elastic modulus $E := 210000 \cdot \frac{N}{mm^2}$ $G := \frac{E}{2 \cdot (1 + 0.3)} = 80769 \cdot \frac{N}{mm^2}$

Member length $L := 6000 \cdot mm$

Coefficients* $C_1 := 1.365$ $C_3 := 0.411$

Critical moment

$$M_{cr} := C_1 \frac{\pi^2 \cdot E \cdot I_z}{L^2} \cdot \left[\sqrt{\frac{I_\omega}{I_z} + \frac{L^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z} + \left(C_3 \cdot z_j\right)^2} + C_3 \cdot z_j\right] = 213.88 kN \cdot m$$

$$F_{cr} := 4 \cdot \frac{M_{cr}}{L} = 142.59 \cdot kN$$

*) G. Sedlacek, J. Naumes: Excerpt from the Background Document to
EN 1993-1-1 Flexural buckling and lateral buckling on a common basis:
Stability assessments according to Eurocode 3 CEN / TC250 / SC3 / N1639E - rev2

B) Computation by ConSteel

- Beam-column FE model (csBeam7)

  Figure 56 shows the LTB of the mono-symmetric member subjected to transverse force. The critical force is computed by the ConSteel software using **csBeam7** finite element model.

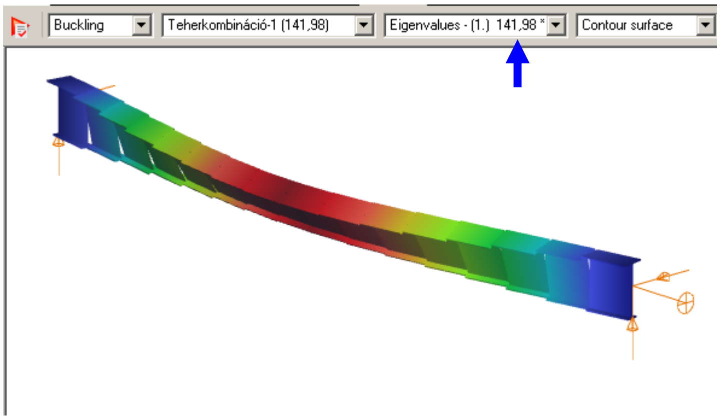


Fig.56 *LTB of simple supported mono-symmetric structural member subjected to transverse force (n=16)*

- Shell FE model (csShell3)

  Figure 57 shows the LTB of the mono-symmetric member subjected to equal end moments. The critical force is computed by the ConSteel software using **csShell3** finite element model.

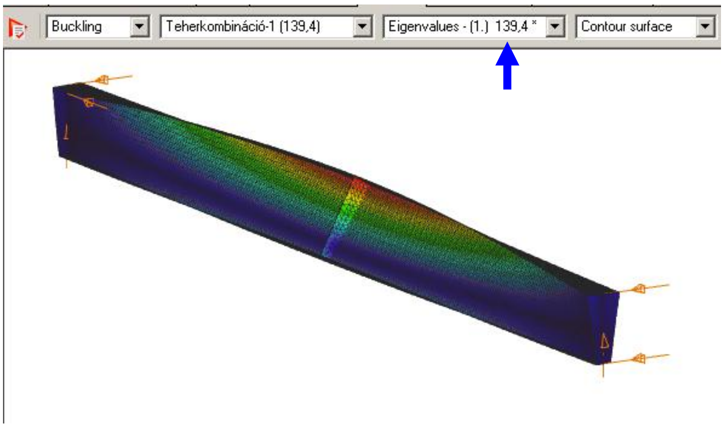


Fig.57 *LTB of simple supported mono-symmetric structural member subjected to transverse force (δ=25mm)*

Evaluation

Table 29 shows the critical moment for lateral torsional buckling of the member which calculated by hand and computed by the ConSteel software using **csBeam7** and **csShell3** models. The result is accurate.

Tab.29 Stability analysis of mono-symmetric member subjected to equal end moments

section	critical force	theory [1]	csBeam7 [2]			csShell3 [3]		
			n	**result**	**1/2**	**δ**	**result**	**1/3**
Welded mono-symmetric I 200-12 ; 400-8 ; 100-12	F_{cr} [kNm]	142,59	2	143,13	**0,996**	50	141,5	**1,008**
			4	142,13	**1,003**	25	139,4	**1,023**
			8*	141,99	**1,004**			
			16	141,98	**1,004**			

*) given by the automatic mesh generation (default)

Note

In the **Table 29 n** denotes the number of the finite elements of the **csBeam7** model, **δ** denotes the maximum shell FE size.

WE-30 Lateral torsional buckling (C section & equal end moments)

Figure 58 shows a simple fork supported member with cold-formed C section (150x100x30x2) subjected to equal end moments. The critical moment is calculated by hand and by the ConSteel software using **csBeam7** model.

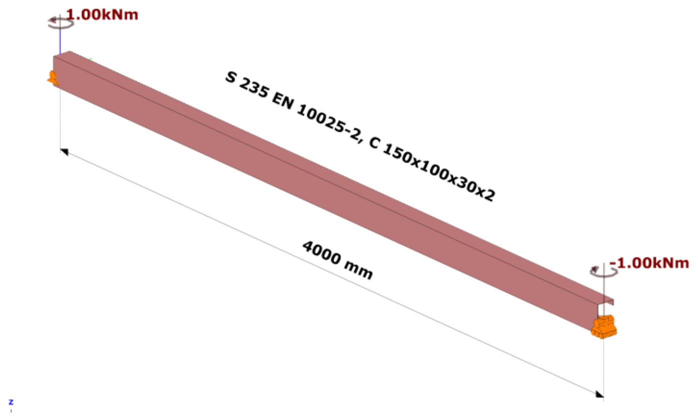


Fig.58 *Simple fork supported member with cold-formed C section subjected to equal and moments (LTB)*

A) Calculation by hand

Section: Cold-formed C section

	width of flange	$b := 100 \cdot mm$
	depth	$d := 150 \cdot mm$
	width of stiffener	$d_1 := 30 \cdot mm$
	plate thickness	$t := 2 \cdot mm$

Cross-sectional properties (by ConSteel GSS model)

$I_y := 3106412 \cdot mm^4$ $\quad I_z := 1206715 \cdot mm^4$

$I_t := 1072 \cdot mm^4$ $\quad I_\omega := 6989423000 \, mm^6$

$e := 38.0 \cdot mm$ $\quad e_s := -60.0 \cdot mm$

Sectional radius*

$$A_f := (d - t) \cdot t = 296 \cdot mm^2$$

$$I_f := \frac{t \cdot (d - t)^3}{12} = 540299 \cdot mm^4$$

$$A_s := \left(d_1 - \frac{t}{2}\right) \cdot t = 58 \cdot mm^2$$

$$I_s := \frac{t \cdot \left(d_1 - \frac{t}{2}\right)^3}{12} + A_s \cdot \left(\frac{d}{2} - \frac{t}{2} - \frac{d_1 - \frac{t}{2}}{2}\right)^2 = 209399 \cdot mm^4$$

$$A_w := \left(b - \frac{t}{2}\right) \cdot t = 198 \cdot mm^2$$

$$I_w := A_w \cdot \left(\frac{d}{2} - \frac{t}{2}\right)^2 = 1084248 \cdot mm^4$$

$$h := b - \frac{t}{2} = 99 \cdot mm$$

$$q_x := \frac{1}{I_z} \cdot \left[e \cdot \left(A_f \cdot e^2 + I_f\right) + 2e_s \cdot \left(A_s \cdot e_s{}^2 + I_s\right) + (2 \cdot e - h) \cdot I_w + \frac{t}{2} \cdot \left[e^4 - (h - e)^4\right]\right] = -41.525 \cdot mm$$

$z_D := 90.9 \cdot mm$

$z_j := z_D - 0.5 q_x = 111.663 \cdot mm$

Length of member $\quad L := 4000 \cdot mm$

Critical moment

$$M_{cr} := \frac{\pi^2 \cdot E \cdot I_y}{L^2} \cdot \left(\sqrt{\frac{I_\omega}{I_y} + \frac{L^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_y} + z_j^2} + z_j\right) = 94.108 kN \cdot m$$

B) Computation by ConSteel

- Beam-column FE model (csBeam7)

  Figure 59 shows the LTB of the member with C section subjected to equal end moments. The critical moment is computed by the ConSteel software using **csBeam7** finite element model.

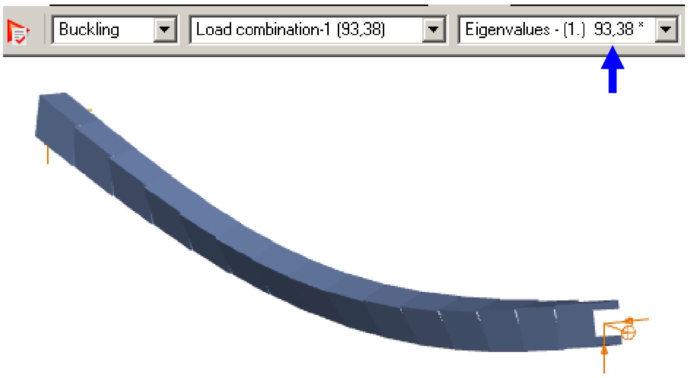


Fig.59 *LTB of simple supported C structural member subjected to equal end moments (n=16)*

Evaluation

Table 30 shows the critical end moment for lateral torsional buckling of the C member calculated by hand and computed by the ConSteel software using **csBeam7** model. The result is accurate.

Tab.30 Stability analysis of the C member subjected ti equal end moments

section	critical force	theory [1]	csBeam7 [2]		
			n	result	1/2
Cold formed C 150x100x30x2	M_{cr} [kNm]	94,108	2	94,07	**0,994**
			4	93,42	**1,007**
			6*	93,38	**1,008**
			16	93,38	**1,008**

*) given by the automatic mesh generation (default)

Note

In the **Table 30** **n** denotes the number of the finite elements of the **csBeam7** model.

WE-31 Lateral torsional buckling (C section & equal end moments)

Figure 60 shows a simple fork supported member with cold-formed C section (150x200x30x2) subjected to equal end moments. The critical moment is calculated by hand and by the ConSteel software using **csBeam7** model.

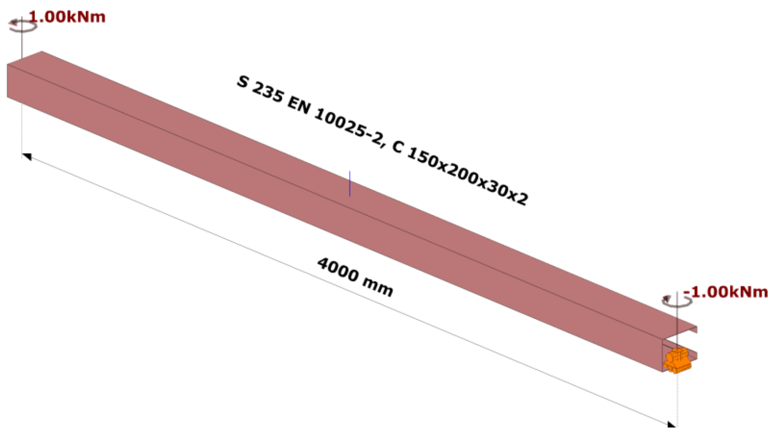


Fig.60 *Simple fork supported member with cold-formed C section subjected to equal and moments (LTB)*

A) Calculation by hand

Section: Cold-formed C section

width of flange	$b := 200 \cdot mm$
depth	$d := 150 \cdot mm$
width of stiffener	$d_1 := 30 \cdot mm$
plate thickness	$t := 2 \cdot mm$

Cross-sectional properties (by ConSteel GSS model)

$I_y := 6362658 \cdot mm^4$ $\quad I_z := 5269945 \cdot mm^4$

$I_t := 1734 \cdot mm^4$ $\quad I_\omega := 35770000000 \cdot mm^6$

$e := 85.2 \cdot mm$ $\quad e_s := -112.8 \cdot mm$

Sectional radius*

$$A_f := (d - t) \cdot t = 296 \cdot mm^2$$

$$I_f := \frac{t \cdot (d - t)^3}{12} = 540299 \cdot mm^4$$

$$A_s := \left(d_1 - \frac{t}{2}\right) \cdot t = 58 \cdot mm^2$$

$$I_s := \frac{t \cdot \left(d_1 - \frac{t}{2}\right)^3}{12} + A_s \cdot \left(\frac{d}{2} - \frac{t}{2} - \frac{d_1 - \frac{t}{2}}{2}\right)^2 = 209399 \cdot mm^4$$

$$A_w := \left(b - \frac{t}{2}\right) \cdot t = 398 \cdot mm^2$$

$$I_w := A_w \cdot \left(\frac{d}{2} - \frac{t}{2}\right)^2 = 2179448 \cdot mm^4$$

$$h := b - \frac{t}{2} = 199 \cdot mm$$

$$q_x := \frac{1}{I_z}\cdot\left[e\cdot\left(A_f\cdot e^2 + I_f\right) + 2e_s\cdot\left(A_s\cdot e_s^{\ 2} + I_s\right) + (2\cdot e - h)\cdot I_w + \frac{t}{2}\cdot\left[e^4 - (h-e)^4\right]\right] = -30.737\cdot mm$$

$$z_D := 187.8\cdot mm$$

$$z_j := z_D - 0.5q_x = 203.168\cdot mm$$

Length of member $L := 4000\cdot mm$

Critical moment $M_{cr} := \frac{\pi^2\cdot E\cdot I_z}{L^2}\cdot\left(\sqrt{\frac{I_\omega}{I_z} + \frac{L^2\cdot G\cdot I_t}{\pi^2\cdot E\cdot I_z} + z_j^{\ 2}} + z_j\right) = 288.68kN\cdot m$

B) Computation by ConSteel

- Beam-column FE model (csBeam7)

Figure 61 shows the LTB of the member with C section subjected to equal end moments. The critical moment is computed by the ConSteel software using **csBeam7** finite element model.

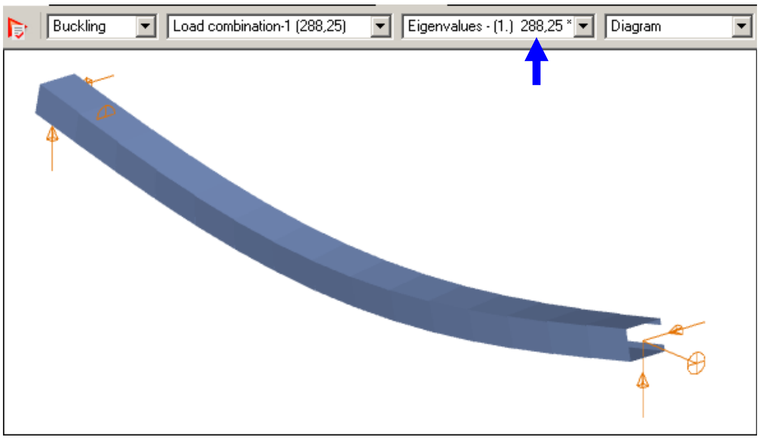


Fig.61 *LTB of simple supported C structural member subjected to equal end moments (n=16)*

Evaluation

Table 31 shows the critical end moment for lateral torsional buckling of the C member calculated by hand and computed by the ConSteel software using **csBeam7** model. The result is accurate.

Tab.31 Stability analysis of the C member subjected ti equal end moments

section	critical force	theory [1]	csBeam7 [2]		
			n	**result**	**1/2**
Cold formed C 150x200x30x2	M_{cr} [kNm]	288,68	2	290,41	**0,994**
			4	288,39	**1,001**
			6*	288,28	**1,001**
			16	288,25	**1,001**

*) given by the automatic mesh generation (default)

Note
In the **Table 31** **n** denotes the number of the finite elements of the **csBeam7** model.

WE-32 Flexural-torsional buckling (U section)

Figure 62 shows a simple fork supported member with cold-formed U section (120x120x4) subjected to compressive force. The critical force is calculated by hand and by the ConSteel software using **csBeam7** and **csShell3** models.

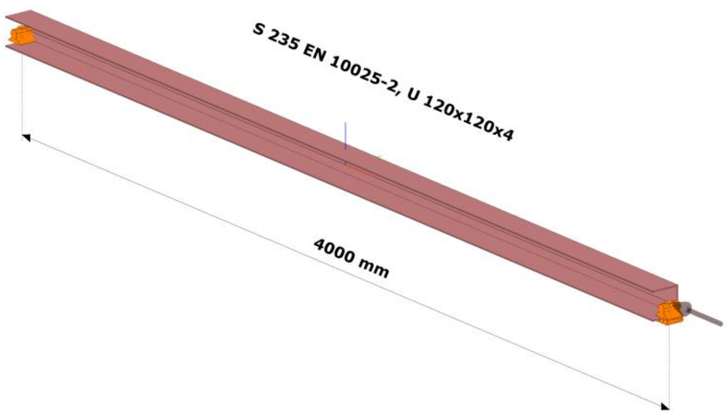


Fig.62 *Simple fork supported member with cold-formed U section subjected to compressive force (FTB)*

A) Calculation by hand

Section: Cold-formed U section

	width of flange	$b := 120 \cdot mm$
	depth	$d := 120 \cdot mm$
	plate thickness	$t := 4 \cdot mm$
Elastic modulus		$E := 210000 \cdot \frac{N}{mm^2}$ $\quad G := \frac{E}{2 \cdot (1 + 0.3)} = 80769 \frac{N}{mm^2}$
Length of member		$L = 4000 \cdot mm$

Cross-sectional properties (by ConSteel GSS model)

$$A := 1408 \cdot mm^2$$

$$I_z := 2180000 \cdot mm^4 \qquad i_z := 39.4 \cdot mm$$

$$I_y := 3699100 \cdot mm^4 \qquad i_y := 51.3 \cdot mm$$

$$I_t := 7927 \cdot mm^4$$

$$I_\omega := 5264600000 \cdot mm^6 \qquad y_\omega := 90.1 \cdot mm$$

$$i_\omega := \sqrt{i_y^2 + i_z^2 + y_\omega^2} = 110.915 \cdot mm$$

$$i_p := \sqrt{\frac{I_y + I_z}{A}} = 64.618 \cdot mm$$

Critical forces

$$P_{cr.y} := \frac{\pi^2 \cdot E \cdot I_y}{L^2} = 479.176 \cdot kN$$

$$P_\omega := \frac{1}{i_\omega^2} \cdot \left(\frac{\pi^2 \cdot E \cdot I_\omega}{L^2} + G \cdot I_t \right) = 107.48 \cdot kN$$

Critical compressive force

$$P_{cr} := \frac{i_\omega^2}{2 \cdot i_p^2} \cdot \left(P_{cr.y} + P_\omega\right) - \sqrt{\frac{i_\omega^4}{4 \cdot i_p^4} \cdot \left(P_{cr.y} + P_\omega\right)^2 - P_{cr.y} \cdot P_\omega \cdot \frac{i_\omega^2}{i_p^2}} = 92.768 \cdot kN$$

B) Computation by ConSteel

- Beam-column FE model (csBeam7)

  Figure 63 shows the flexural torsional buckling of the member with U section subjected to compressive force. The critical force is computed by the ConSteel software using **csBeam7** finite element model.

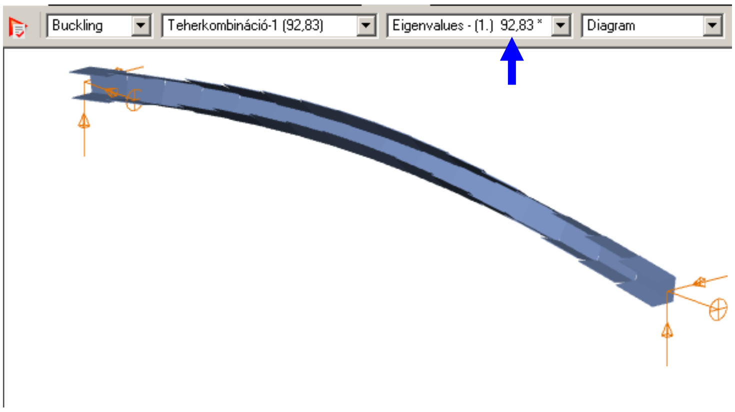


Fig.63 *FTB of the simple supported U structural member subjected to compressive force (n=16)*

- Shell FE model (csShell3)

  Figure 64 shows flexural torsional buckling of the member with U section subjected to compressive force. The critical force is computed by the ConSteel software using **csShell3** finite element model.

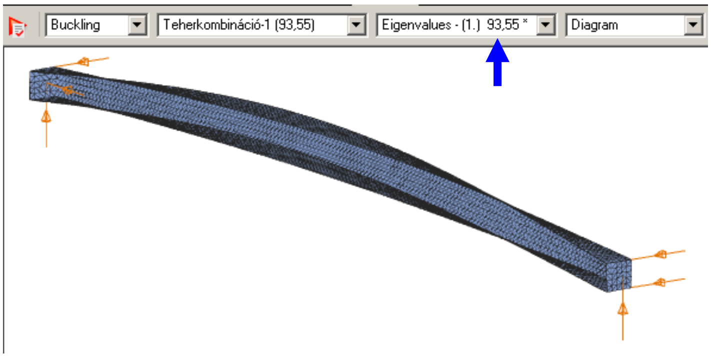


Fig.64 *FTB of the simple supported U structural member subjected to compressive force (δ=25mm)*

Evaluation

Table 32 shows the critical compressive force for flexural lateral buckling of the member which calculated by hand and computed by the ConSteel software using both of the **csBeam7** and **csShell3** models. The results are accurate.

Tab.32 Stability analysis of member subjected to compressive force

section	critical force	theory [1]	ConSteel					
			csBeam7 [2]			csShell3 [3]		
			n	result	1/2	δ	result	1/3
U 120x120x4 cold formed	P_{cr} [kN]	92,77	2	93,24	0,995	50	94,42	0,983
			4	92,86	0,999	25	93,55	0,992
			6*	92,84	0,999			
			16	92,83	0,999			

*) given by the automatic mesh generation (default)

Notes

In the **Table 32** **n** denotes the number of the finite elements of the **csBeam7** model, **δ** denotes the maximum size of the shell finite elements in the **csShell3** model in [mm].

WE-33 Interaction of flexural buckling and LTB (symmetric I section & equal end moments and compressive force)

Figure 65 shows a simple fork supported member with welded symmetric I section (200-12, 400-8) subjected to compressive force and equal end moments. The critical moment with constant compressive force is calculated by hand and by the ConSteel software using **csBeam7** model.

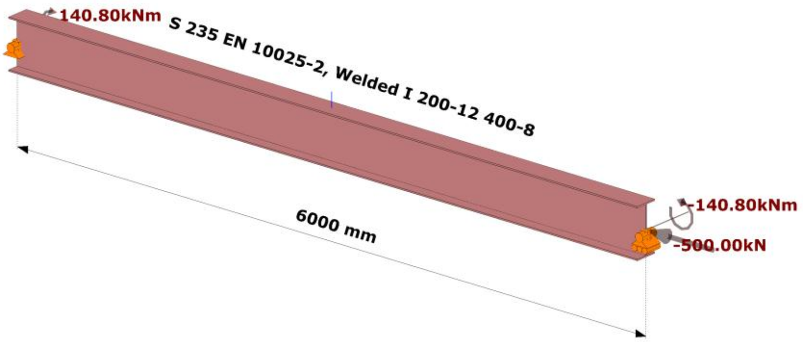


Fig.65 *Simple fork supported member with welded I section subjected to constant compressive force and equal end moments (interaction)*

A) Calculation by hand

Section: welded symmetric I section

flange $b := 200 \cdot mm$ $t_f := 12 \cdot mm$

web $h_w := 400 \cdot mm$ $t_w := 8 \cdot mm$

Sectional properties (by GSS model)

$A := 8000 \cdot mm^2$

$I_y := 246417000\ mm^4$ $i_y := 175.5 \cdot mm$

$I_z := 16017000\ mm^4$ $i_z := 44.7 \cdot mm$

$I_t := 301351 \cdot mm^4$

$I_\omega := 678.2 10^9 \cdot mm^6$ $i_\omega := \sqrt{i_y^2 + i_z^2} = 181.103mm$

Elastic modulus $E := 210000 \cdot \frac{N}{mm^2}$ $G := \frac{E}{2 \cdot (1 + 0.3)} = 80769 \cdot \frac{N}{mm^2}$

Member length $L := 6000 \cdot mm$

Critical forces

$$P_{cr.z} := \frac{\pi^2 \cdot E \cdot I_z}{L^2} = 922.142 \cdot kN$$

$$P_\omega := \frac{1}{i_\omega^2} \cdot \left(G \cdot I_t + \frac{\pi^2 \cdot E \cdot I_\omega}{L^2} \right) = 1932.588\ kN$$

$$M_{cr} := \frac{\pi^2 \cdot E \cdot I_z}{L^2} \cdot \sqrt{\frac{I_\omega}{I_z} + \frac{L^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z}} = 241.766kN \cdot m$$

Critical moment with constatnt compressive force

$$P := 500 \cdot kN \qquad M := M_{cr} \cdot \sqrt{\left(1 - \frac{P}{P_{cr.z}}\right) \cdot \left(1 - \frac{P}{P_\omega}\right)} = 140.8kN \cdot m$$

B) Computation by ConSteel

- Beam-column FE model (csBeam7)

Figure 66 shows the interactive buckling of the member with welded I section subjected to constant compressive force and equal end moments. The critical moment is computed by the ConSteel software using **csBeam7** finite element model.

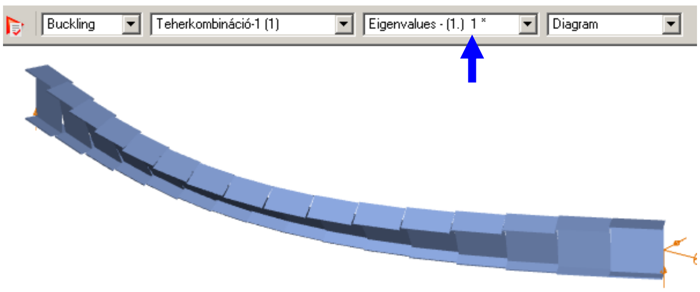


Fig.66 *Interactive buckling of the simple supported structural member subjected to constant compressive force and equal end moments (n=16)*

Evaluation

Table 33 shows the critical moment for the interactive buckling mode of the member subjected to constant compressive force (P=500kN) and equal end moments. The cririical moment was calculated by hand and computed by the ConSteel software using **csBeam7** model. The result is accurate.

Tab.33 Stability analysis of the member subjected to constant compressive force and equal end moments

section	critical moment (P=500 kN)	theory [1]	csBeam7 [2]		
			n	result	1/2
Welded I 200-12 ; 400-8	M_{cr} [kNm]	140,8	2	142,0	0,992
			4	140,8	1,000
			6*	140,8	1,000
			16	140,8	1,000

*) given by the automatic mesh generation (default)

Notes

In the **Table 33** **n** denotes the number of the finite elements of the **csBeam7** model.

3. Design

3.1 Simple members

The following two worked examples (WE-34 & WE-35) were published in the following paper:

HUGHES, A.F., ILES, D.C. and MALIK, A.S.: Design of steel beams in torsion, SCI Publication P385, In accordance with Eurocodes and the UK National Annexes, p. 96 (Example 1 & 2)

WE-34: Unrestrained beam with eccentric point load

A simply supported beam spans 4 m without intermediate restraint (see **Figure 67**). It is subject to a permanent concentrated load of 74 kN at mid-span, which is attached to the bottom flange at an eccentricity of 75 mm. Verify the trial section 254UKC73 (S275). Any restraint provided by the end plate connections against warping is partial, unreliable and unquantifiable. The ends of the member will therefore be assumed to be free to warp.

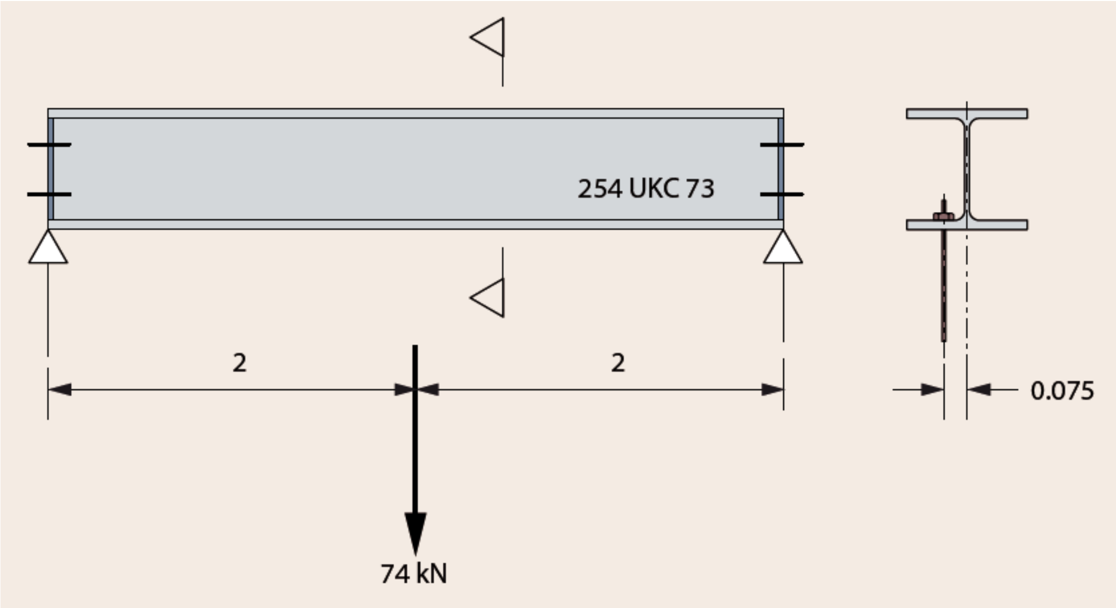


Fig.67 Unrestrained beam with eccentric point load

Section properties

properties	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
A	9.310 mm^2	9.323 mm^2	**0,999**
I_z	39.100.000 mm^4	39.079.227 mm^4	**1,000**
$W_{pl.y}$	992.000 mm^3	992.909 mm^{3*}	**0,999**
$W_{pl.z}$	465.000 mm^3	460.230 mm$^{3\,*}$	**1,010**
I_T	576.000 mm^4	591.937 mm^4	**0,973**
I_w	562.000.000.000 mm^6	556.700.000.000 mm^6	**1,010**
f_y	275 N/mm^2	275 N/mm$^{2\,**}$	**1,000**

*) by EPS model (approximation)
**) by EN 1993-1-1

Design values of vertical and horizontal bending moments and shear

internal force *	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
$M_{y.Ed}$	102 kNm	103,2 kNm	**0,988**
V_{Ed}	52 kNm	52,56 kNm	**0,989**

*) by first order theory

Maximum rotation of the beam

position	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
mid-span	0,053 rad	0,052 rad	**1,019**

Total (second order) minor axis bending

internal force	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
$M_{z.Ed}$	5,4 kNm *	5,010 kNm **	**1,078**

*) approximation
**) 'exact' numerical result by second order analysis

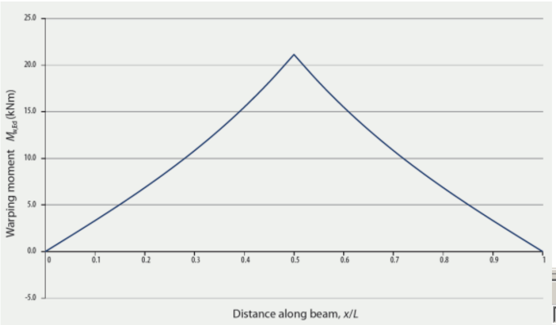
Warping moment

internal force	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
$M_{w.Ed.max}$	21.1 kNm *	19.77 kNm **	**1,067**

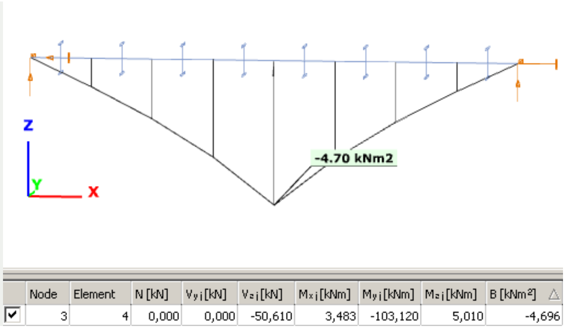
*) for one flange

**) calculated from B bimoment: $M_w = \frac{1}{2} B \frac{I_z}{I_\omega} \frac{\omega}{y_{max}}$

SCI Publication P385



ConSteel



Cross-sectional resistance

resistance	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
$M_{y.Rd}$	273 kNm	273.1 kNm	**1,000**
$M_{z.Rd}$	128 kNm	126,6 kNm	**1,011**
$V_{pl.Rd}$	406 kN	406,8 kN	**0,998**

Bending resistance

used resistance	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
η	0,51 *	0,988 ** (0,421 ***)	**0,516** **(1,210)**

*) non-linear plastic interaction formula of UK Annex
**) elastic resistance formula of EC3-1-1 with warping effect (6.2.1 (5))
***) plastic interaction formula of EC3-1-1 neglecting warping effect (6.2.1 (7))

Buckling resistance

property	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
M_{cr}	1.049 kNm *	1.062 kNm ** (1.632 kNm ***)	**0,999**
λ_{LT}	0,51	0,507	**0,981**
χ_{LT}	0,950	0,957	**1,028**
$M_{b.Rd}$	259 kNm	273,1 kNm ****	**0,986**

*) computed by LTBeam software
**) force acts in centroid
***) force acts on bottom flange (basic condition of the example)
****) with f_y=275N/mm^2 (EC3-1-1)

Interaction between LTB, minor axis bending and torsion effects

used capacity	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
η	0,66 *	0,419 **	**1,575**

*) by the special formula specified by UK National Annex for EN 1993-1-1
**) by the General Method EN 1993-1-1 6.3.4 with M_{cr} taken eccentricity into consideration but neglecting the effect of warping moment

Evaluation

The worked example of **SCI Publication P385 Example 1** is a hand design oriented example using approximations to take torsional behavior and second order effects into consideration. Interaction design between LTB, minor axis bending and torsion effects was calculated by the special formula specified by the UK National Annex for EN1993-1-1. ConSteel software uses exact numerical solution for torsion and second order effect. ConSteel uses the General Method of EN 1993-1-1 for interaction buckling design which neglects the effect of warping in the design. ConSteel uses elastic cross-section resistance formula taking the warping effect into consideration. However, the design by UK Annex leads to considerable higher resistance than the EC3-1-1 (58%).

WE-35: Crane beam subject to two wheel loads

A crane beam spans 7.5 m without intermediate restraint (see **Figure 68**). Verify the chosen 533 × 210 UKB 101 section under the condition shown below, in which two wheel loads 3 m apart act at rail level 65 mm above the beam. The ULS design values of the loads from the wheels of the crane are 50 kN vertical together with 3 kN horizontal. Allow 2 kN/m for the design value of the self weight of the beam and crane rail. Consider the design effects for the location shown below (which gives maximum vertical bending moment). Assume that an elastomeric pad will be provided between the rail and the beam. According to EN 1993-6, 6.3.2.2(2), the vertical wheel reaction should then be taken as being effectively applied at the level of the top of the flange and the horizontal load at the level of the rail.

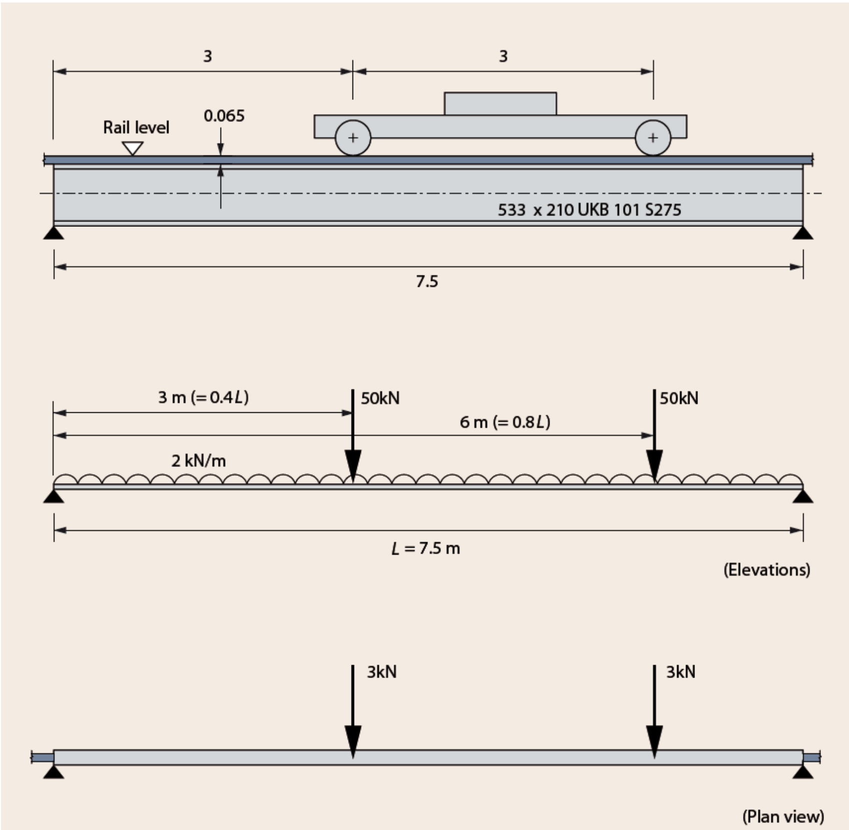


Fig.68 Crane beam subject to two wheel loads

Section properties

properties	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
A	12.900 mm^2	12.867 mm^2	**1,003**
I_z	26.800.000 mm^4	26.857.000 mm^4	**0,998**
$W_{pl.y}$	2.610.000 mm^3	2.613.112 mm^3	**0,999**
$W_{pl.z}$	399.000 mm^3	383.670 mm^3 *	**1,040**
I_T	1.010.000 mm^4	1.016.404 mm^4	**0,994**
I_w	1.810.000.000.000 mm^6	1.811.000.000.000 mm^6	**1,000**
f_y	265 N/mm^2	275 N/mm^2 **	**0,964**

**) by EPS model (approximation)
***) by EN 1993-1-1

Design values of vertical and horizontal bending moments and shear

internal force *	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
$M_{y.Ed}$	133,5 kNm	133,5 kNm	**1,000**
$M_{z.Ed}$	7,2 kNm	7,2 kNm	**1,000**

*) by first order theory

Maximum rotation of the beam

position	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
LH wheel	0,84 deg	0,834 deg	**1,007**
maximum *	-	0,876 deg	-

*) not given by the publication

Total (second order) minor axis bending

internal force	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
$M_{z.Ed}$	9,2 kNm *	10,87 kNm **	**0,846**

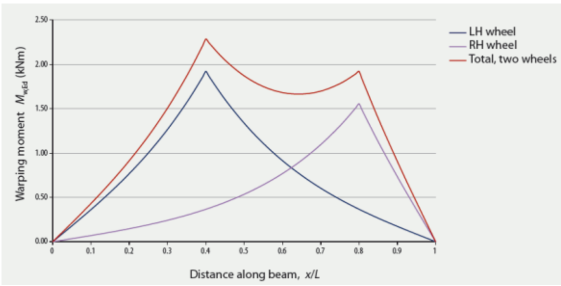
*) approximation
**) 'exact' numerical result

Warping moment

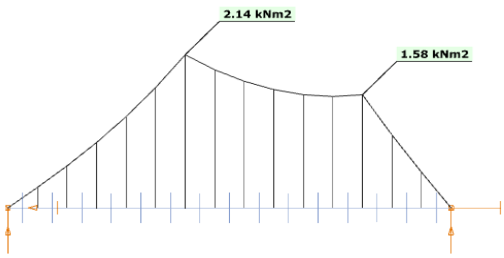
internal force	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
$M_{w.Ed.max}$	2,28 kNm *	2,14 kNm^2 **	**1,065**

*) for one flange
**) bimoment

SCI Publication P385



ConSteel



Cross-sectional resistance

resistance	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
$M_{y.Rd}$	692 kNm	718,6 kNm *	**0,963**
$M_{z.Rd}$	106 kNm	105,5 kNm *	**1,005**
$V_{pl.Rd}$	952 kN	982,6 kN *	**0,967**

*) calculated with f_y=275N/mm^2

Buckling resistance

property	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
M_{cr}	320 kNm	320,4 kNm	**0,999**
λ_{LT}	1,47	1,498	**0,981**
χ_{LT}	0,401	0,39	**1,028**
$M_{b.Rd}$	277 kNm	280,5 kNm	**0,986**

Interaction between LTB, minor axis bending and torsion effects

used capacity	SCI Publication P385 [1]	ConSteel (csBeam7 model) [2]	1/2
η	0,62 *	0,579 **	**1,071**

*) by the special formula specified by UK National Annex for EN 1993-1-1
**) by the General Method EN 1993-1-1 6.3.4

Evaluation

The worked example of **SCI Publication P385 Example 2** is a hand design oriented example using approximations to take torsional behavior and second order effects into consideration. Interaction design between LTB, minor axis bending and torsion effects is calculated by the special formula specified by UK National Annex for EN1993-1-1. Contrary, the ConSteel software uses exact numerical solution for torsion and second order effect and it uses the General Method of EN 1993-1-1 for interaction buckling design (neglecting the effect of torsion). However, the deviation in the governing result of the design by the two approaches is not more than 7%.

WE-36 Simply supported beam with lateral restraint at load application point

Figure 69 shows a Simply supported beam with lateral restraint at load application point

A) Verification

Access Steel example (SX007): Simply supported beam with lateral restraint at load application point

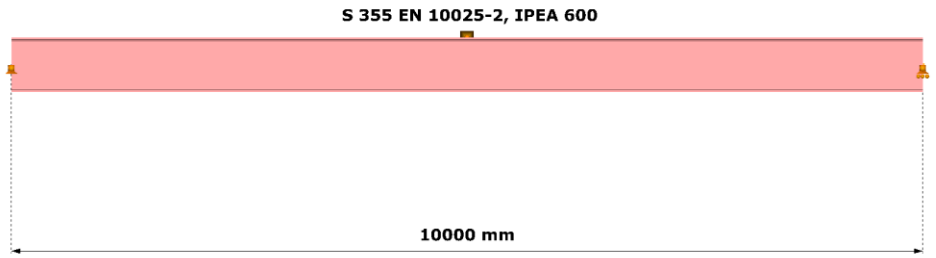
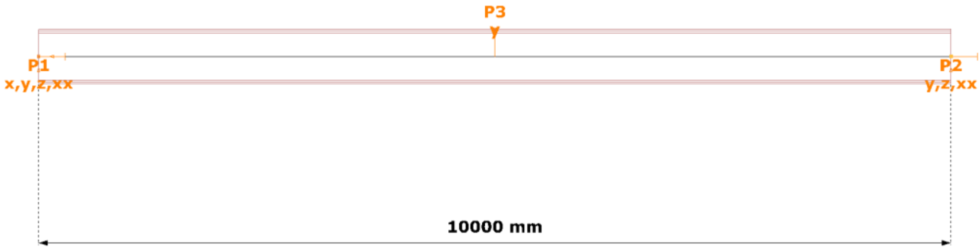
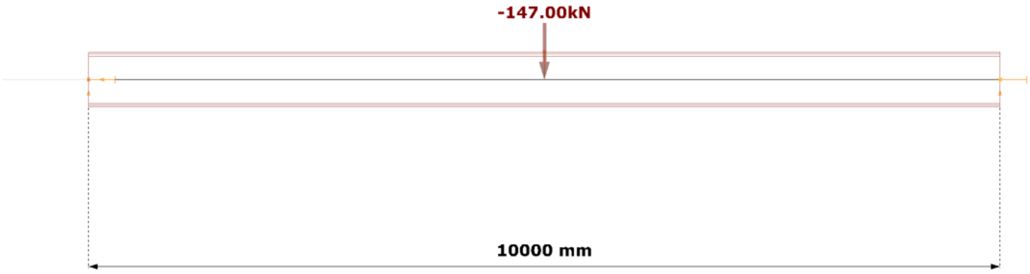


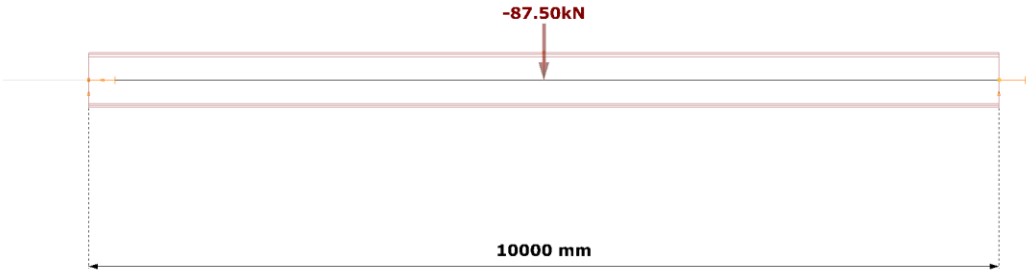
Fig.69 Simply supported beam with lateral restraint at load application point



Loads
Permanent loads
Self weight of the beam +147kN at 5000mm



Imposed loads

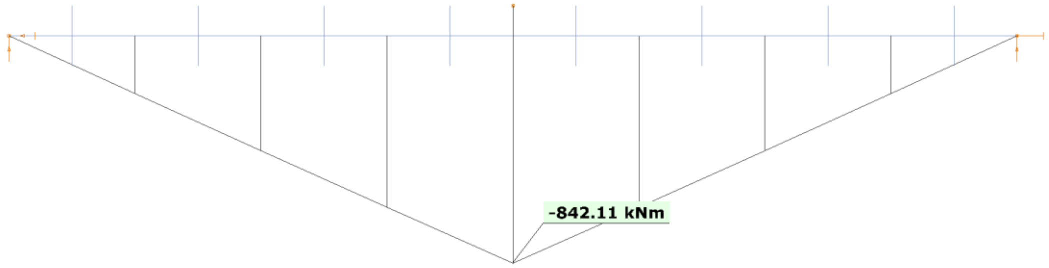


Load combinations

Name	Limit state	Self Weight	Permanent load	Imposed load
Load combination-1	(ULS) Ultimate	1,35	1,35	1,5

Analysis results

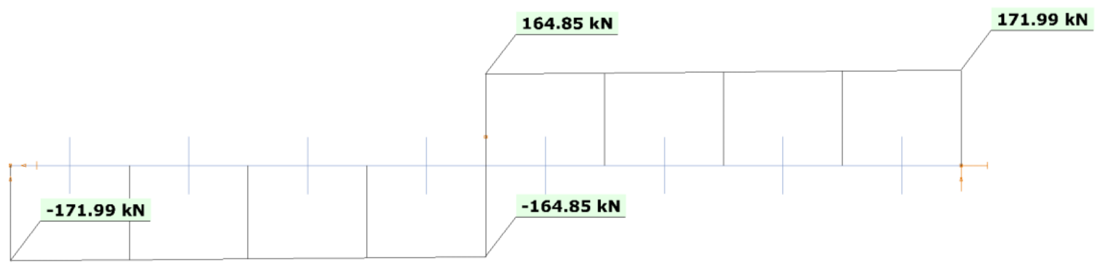
Moment diagram – Load Combination 1



Bending moment value at midspan (5000 mm)

Load combination	ConSteel first order analysis results [kNm]	Reference value[kNm]	Difference[%]
Combination 1	-842,11	-842,13	0,0

Shear diagram – Load Combination 1

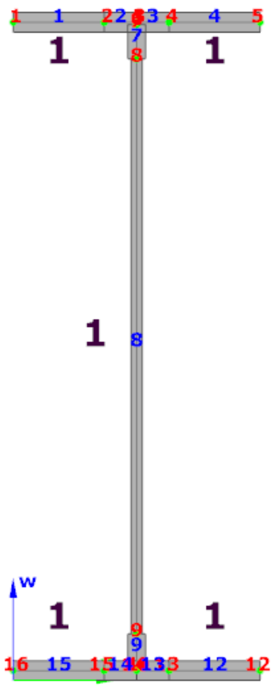


Dominant shear force

Load combination	ConSteel first order analysis results [kN]	Reference value[kN]	Difference[%]
Combination 1	-171,99	172	0,0

Beam verification

Section classification

<table>
<tr><td>ConSteel results</td><td>Reference result</td></tr>
<tr><td>
</td><td>All plates are class 1</td></tr>
</table>

Cross section resistance check

Bending about the major axis

<table>
<tr><td>ConSteel results</td><td>Reference results</td><td>Difference[%]</td></tr>
<tr><td>
<table>
<tr><td>Bending about the major axis</td><td></td></tr>
<tr><td>Utilization</td><td>75,5 %</td></tr>
<tr><td>Section class</td><td>1</td></tr>
<tr><td>Applied part of standard</td><td>6.2.5 (1)-(3) - (6.12-6.15) formula</td></tr>
<tr><td>$M_{y,Ed}$</td><td>-842,1 kNm</td></tr>
<tr><td>$M_{y,c,Rd}$</td><td>1 115,1 kNm</td></tr>
<tr><td>$W_{pl,y,min}$</td><td>3 141 180,8 mm^3</td></tr>
<tr><td>f_y</td><td>355,0 N/mm^2</td></tr>
<tr><td>γ_{M0}</td><td>1,0</td></tr>
</table>
</td><td>$M_{c,Rd}$=1115 kN</td><td>0,0</td></tr>
</table>

Minor axis shear

ConSteel results		Reference results	Difference[%]
Minor axis shear			
Utilization	11,5 %		
Section class	1		
Applied part of standard	6.2.6 (1)-(3) - (6.17, 6.18) formula		
$V_{z,Ed}$	164,8 kN		
$V_{z,c,Rd}$	1 437,5 kN	V_{Rd}=1437 kN	0,0
A_z	7 013,5 mm²		
f_y	355,0 N/mm²		
γ_{M0}	1,0		

Stability check of the beam

Lateral torsional buckling

ConSteel results		Reference results	Difference[%]
Summary of results			
Used capacity in lateral-torsional buckling:	89,5%		
Place of the dominant cross section:	5000 mm from the first n		
Number of the dominant finite element:	4		
Place of the dominant FE node:	k		
Class of the dominant cross section for compres	1		
Used part of standard:	6.3.2(6.46-6.49) formula		
Detailed calculation			
$M_{y,Ed}$	842,1 kNm		
$M_{y,b,Rd}$	940,9 kNm	M_{cr}=942,2 kNm	0,1
M_{cr}	1585,3 kNm	M_{cr}=1590 kNm	0,3
L	5000 mm		
k	1,000		
k_w	1,000		
C_1	1,770		
C_2	1,000		
C_3	0,939		
z_g	0 mm		
λ_{LT}	0,839	λ_{LT}=0,837	0,2
α_{LT}	0,490		
Φ	0,871		
χ_{LT}	0,739	χ_{LT}=0,740	0,0
$\chi_{LT,mod}$	0,844		
f	0,876	f=0,876	0,0
k_c	0,752	k_c=0,752	0,0
$W_{pl,y}$	3141180,8 mm³		
f_y	355,0 N/mm²		
γ_{M1}	1,0		

WE-37 Simply supported laterally unrestrained beam

Figure 70 shows a simply supported beam.

A) Verification

Acces Steel example (SX001): [Simply supported laterally unrestrained beam]

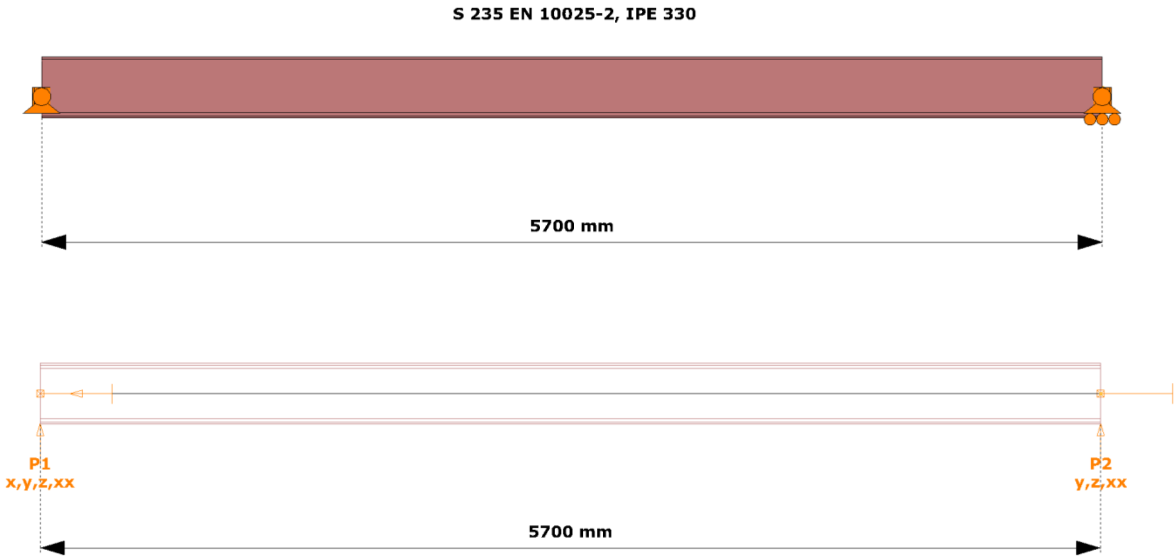
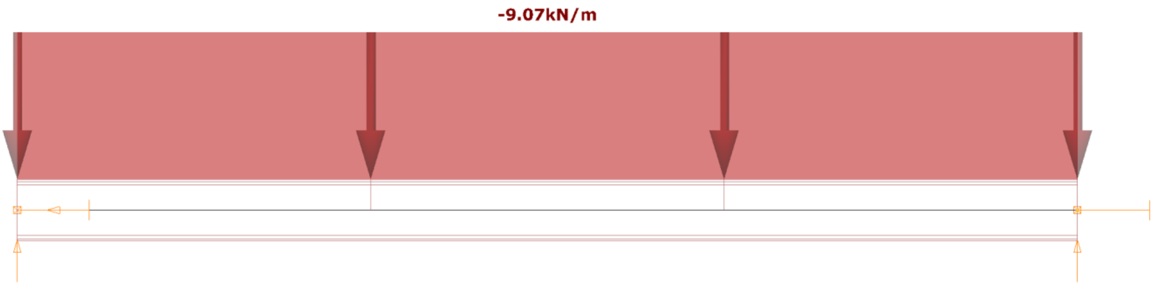


Fig.70 Simply supported beam

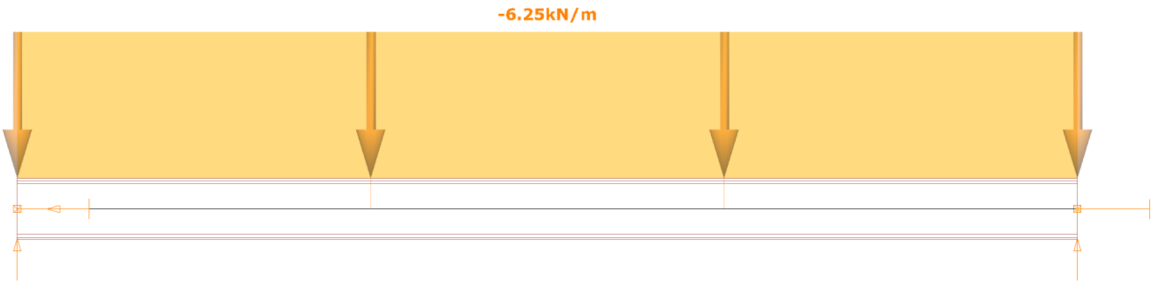
Loads

Permanent loads

Self weight of the beam is calculated by ConSteel



Imposed loads

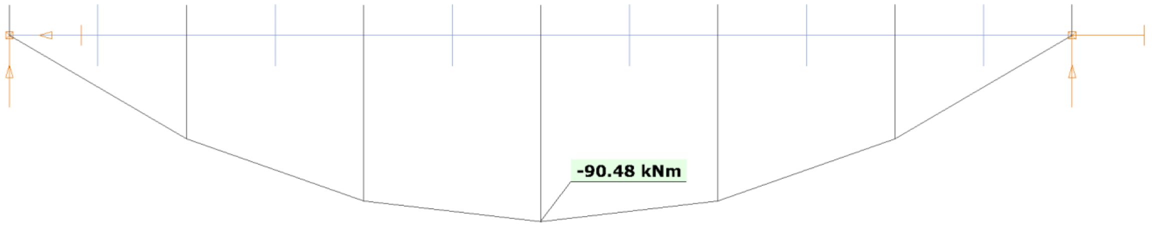


Load combinations

Name	Limit state	DL	LL
Load combination-1	(ULS) Ultimate	1,35	1,5

Analysis results

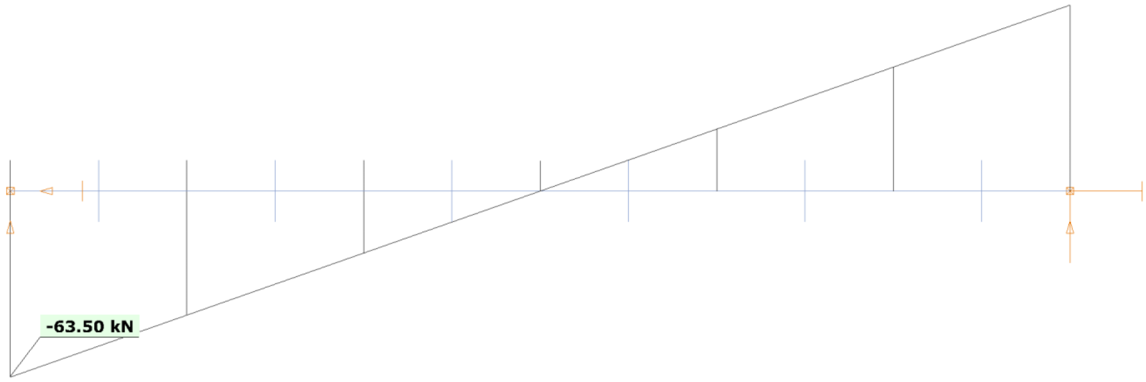
Moment diagram – Load Combination 1



Bending moment value at midspan (2850 mm)

Load combination	ConSteel first order analysis results [kNm]	Reference value[kNm]	Difference[%]
Combination 1	-90,48	-90,48	0,0

Shear diagram – Load Combination 1

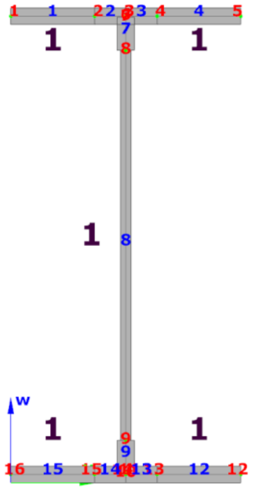


Dominant shear force

Load combination	ConSteel first order analysis results [kN]	Reference value[kN]	Difference[%]
Combination 1	-63,50	-63,50	0,0

Beam verification

Section classification

ConSteel results	Reference result
	All plates are class 1

Cross section resistance check

Bending about the major axis

ConSteel results		Reference results	Difference[%]
Bending about the majc			
Utilization	47,9 %	47,9	0,0
Section class	1		
Applied part of stanc	6.2.5 (1)-(3) - (6.12-6.15) formula		
$M_{y,Ed}$	-90,5 kNm		
$M_{y,c,Rd}$	189,0 kNm	$M_{c,Rd}$=189,01 kN	0,0
$W_{pl,y,min}$	804 330,7 mm³		
f_y	235,0 N/mm²		
γ_{M0}	1,0		

Minor axis shear

ConSteel results		Reference results	Difference[%]
Minor axis shear			
Utilization	15,2 %	15,2	0,0
Section class	1		
Applied part of stanc	6.2.6 (1)-(3) - (6.17, 6.18) formula		
$V_{z,Ed}$	-63,5 kN		
$V_{z,c,Rd}$	418,0 kN	$V_{pl,z,Rd}$=417,9 kN	0,02
A_z	3 080,9 mm²		
f_y	235,0 N/mm²		
γ_{M0}	1,0		

Stability check of the beam

Lateral-torsional buckling

ConSteel results		Reference results	Difference[%]
Lateral-torsional buckling			
Summary of results			
Used capacity in lateral-torsional buckling:	98,2%	98,1	0,10
Place of the dominant cross section:	2850 mm from the first n		
Number of the dominant finite element:	3		
Place of the dominant FE node:	k		
Class of the dominant cross section for compress	1		
Used part of standard:	6.3.2(6.46-6.49) formula		
Detailed calculation			
$M_{y,Ed}$	90,5 kNm		
$M_{y,b,Rd}$	92,2 kNm	$M_{b,Rd}$=92,24kNm	0,04
M_{cr}	113,8 kNm	M_{cr}=113,9 kNm	0,09
L	5700 mm		
k	1,000		
k_w	1,000		
C_1	1,132		
C_2	0,459		
C_3	0,525		
z_g	165 mm		
λ_{LT}	1,289	λ_{LT}=1,288	0,08
α_{LT}	0,490		
Φ	1,341	ϕ=1,34	0,0
χ_{LT}	0,480	χ_{LT}=0,480	0,0
$\chi_{LT,mod}$	0,488	χ_{LTmod}=0,488	0,0
f	0,984	f=0,984	0,0
k_c	0,940	k_c=0,94	0,0
$W_{pl.y}$	804330,7 mm^3		
f_y	235,0 N/mm^2		
γ_{M1}	1,0		

WE-38 Simply supported beam with continuous lateral and twist restraint

Figure 71 shows a simply supported beam. The beam is continuously braced against lateral deflections and twist rotations.

A) Verification

The Behaviour and Design of Steel Structure to EC3 (fourth edition): 7.7.2 Example 2

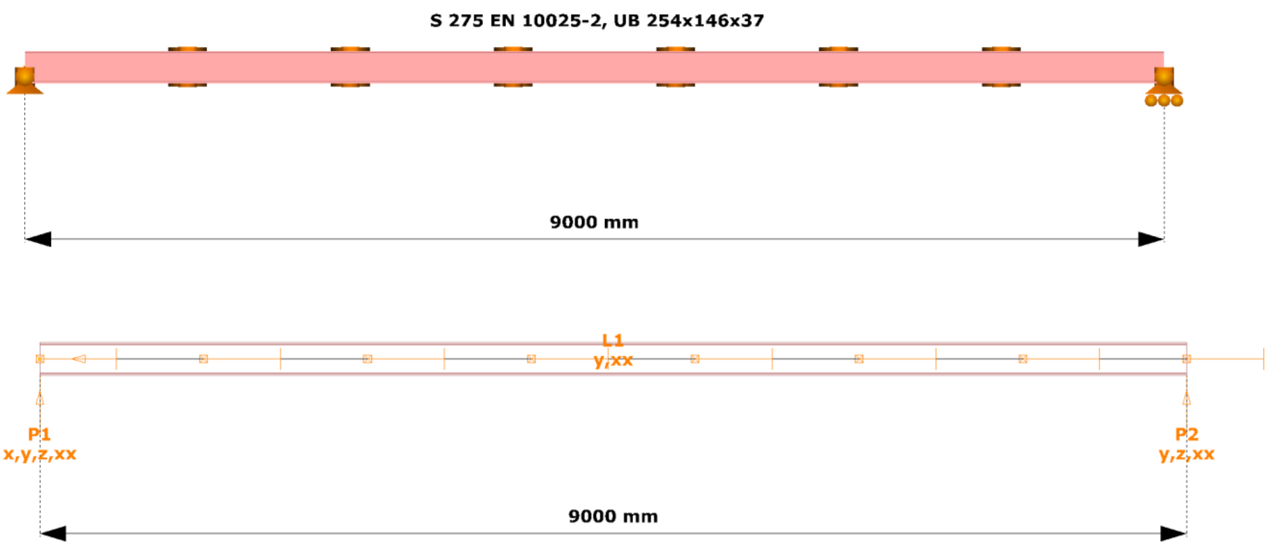
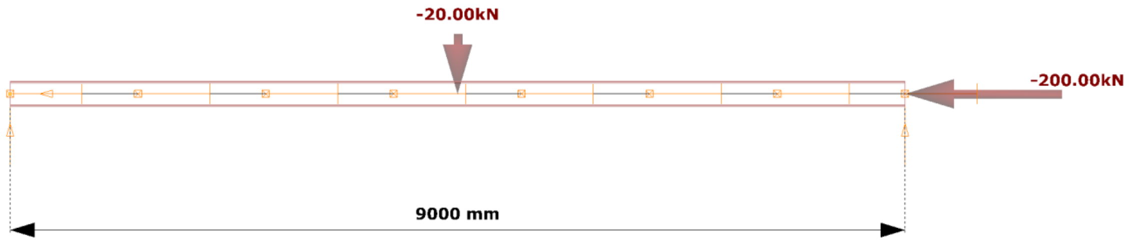


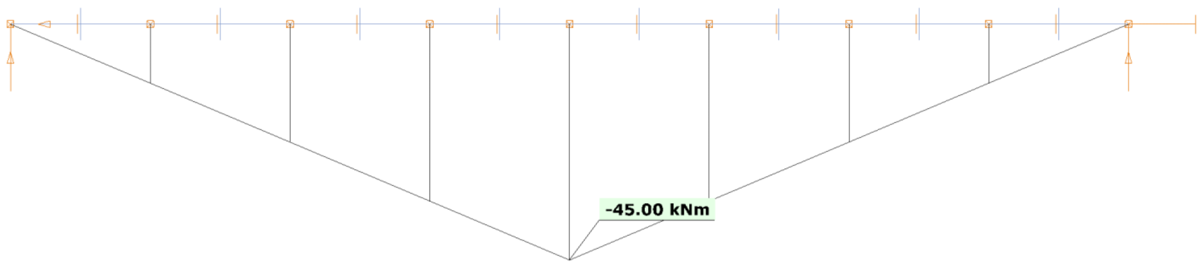
Fig.71 *Simply supported beam with continuous lateral and twist restraint*

Loads



Analysis results

Moment diagram

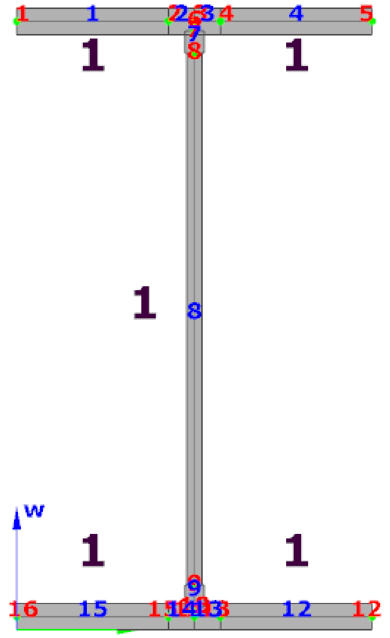


Bending moment value at midspan (4500 mm)

Load combination	ConSteel first order analysis results [kNm]	Reference value[kNm]	Difference[%]
Combination 1	-45,00	-45,00	0,0

Beam verification

Section classification

ConSteel results	Reference result
	All plates are class 1

Cross section resistance check

Compression

<table>
<tr><th colspan="2">ConSteel results</th><th>Reference results</th><th>Difference[%]</th></tr>
<tr><td>Compression</td><td></td><td rowspan="9">No results</td><td rowspan="9"></td></tr>
<tr><td>Utilization</td><td>15,4 %</td></tr>
<tr><td>Section class</td><td>1</td></tr>
<tr><td>Applied part of stand</td><td>6.2.4 (1)-(2) - (6.9-6.11) formula</td></tr>
<tr><td>N_{Ed}</td><td>-200,0 kN</td></tr>
<tr><td>$N_{c,Rd}$</td><td>1 297,1 kN</td></tr>
<tr><td>A</td><td>4 716,6 mm²</td></tr>
<tr><td>f_y</td><td>275,0 N/mm²</td></tr>
<tr><td>γ_{M0}</td><td>1,0</td></tr>
</table>

Bending about the major axis

ConSteel results		Reference results	Difference[%]
Bending about the maj		$M_{c,Rd}$=132,8 kN	0,0
Utilization	33,9 %		
Section class	1		
Applied part of stand	6.2.5 (1)-(3) - (6.12-6.15) formula		
$M_{y,Ed}$	-45,0 kNm		
$M_{y,c,Rd}$	132,9 kNm		
$W_{pl,y,min}$	483 230,8 mm³		
f_y	275,0 N/mm²		
γ_{M0}	1,0		

Minor axis shear

ConSteel results		Reference results	Difference[%]
Minor axis shear		No results	
Utilization	3,6 %		
Section class	1		
Applied part of stand	6.2.6 (1)-(3) - (6.17, 6.18) formula		
$V_{z,Ed}$	-10,0 kN		
$V_{z,c,Rd}$	279,3 kN		
A_z	1 759,4 mm²		
f_y	275,0 N/mm²		
γ_{M0}	1,0		

Stability check of the beam

Strong axis buckling

ConSteel results		Reference results	Difference[%]
N_{Ed}	200,0 kN		
$N_{b,Rd}$	902,6 kN	$N_{b,rd}$=900kN	0,29
N_{cr}	1418,0 kN		
L	9000 mm		
k	0,999		
λ	0,956	λ=0,960	-0,42
α	0,210		
Φ	1,037	ϕ=1,041	-0,38
χ	0,696	χ=0,693	0,43
A	4716,6 mm²		
f_y	275,0 N/mm²		
γ_{M1}	1,0		

Interaction of buckling and bending

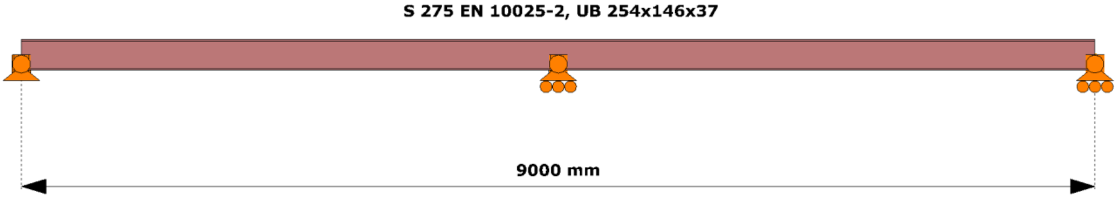
ConSteel results		Reference results	Difference[%]
Summary of results			
Used capacity	57,7%	57,9%	0,35
Used part of standard:	6.3.3 (6.61-6.62) formula		
Applied method for interaction factors	Method 2 (Annex B)		
Strong axis buckling			
Summary of results			
Used capacity in lateral buckling:	22,2%		
Place of the dominant cross section:	0 mm from the first node		
Number of the dominant finite element:	1		
Place of the dominant FE node:	j		
Class of the dominant cross section for compres	2		
Used part of standard:	6.3.1(6.46-6.49) formula		
Detailed calculation			
Results of major axis bending			
Capacity of major axis bending check:	33,9%		
Place of the dominant cross section:	4500 mm from the first n		
Number of the dominant finite element:	4		
Place of the dominant FE node:	j		
Class of the dominant cross section for bending:	1		
Used part of standard:	6.2.5 (6.12-6.15) formula		
$M_{y,Ed}$	45,0 kNm		
$M_{y,b,Rd}$	132,9 kNm		
$W_{pl.y}$	483230,8 mm^3		
f_y	275,0 N/mm^2		
γ_{M1}	1,0		
Interaction factors			
Used part of standard:	Annex B, Table B1-B3		
k_{yy}	1,051	k_{yy}=1,052	0,0
C_{my}	0,900	C_{my}=0,90	0,0

WE-39 Two span beam

Figure 72 shows a two span beam. The beam is braced against lateral deflections and twist rotations in the middle.

A) Verification

The Behaviour and Design of Steel Structure to EC3 (fourth edition): 7.7.3 Example 3



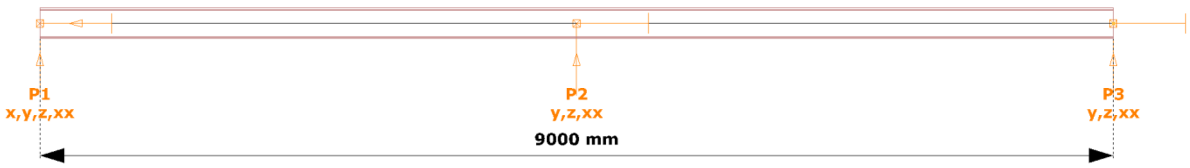
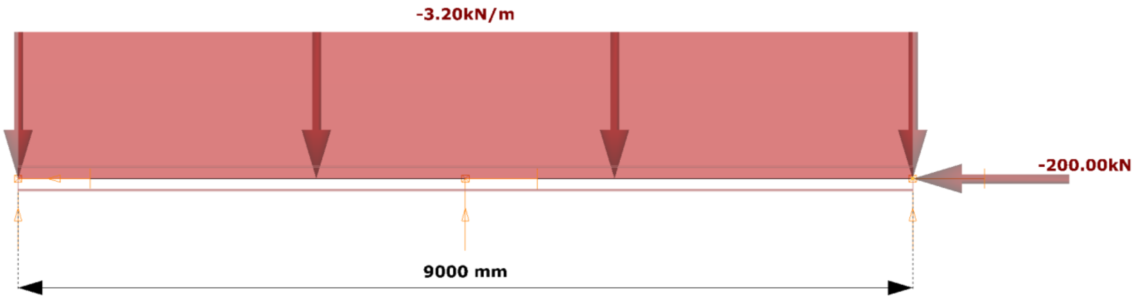


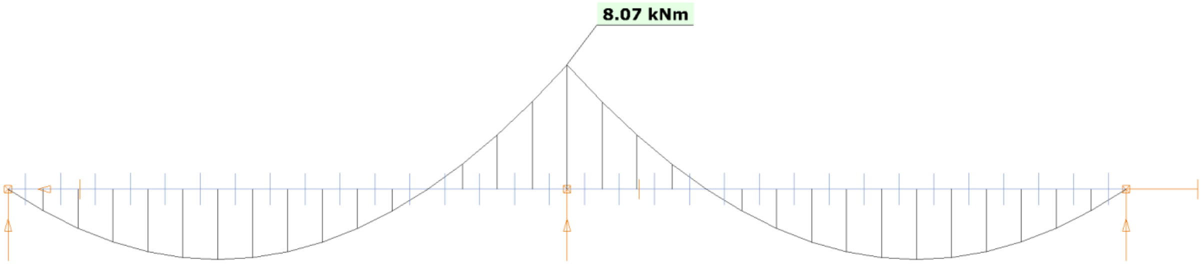
Fig.72 Two span beam

Loads



Analysis results

Moment diagram

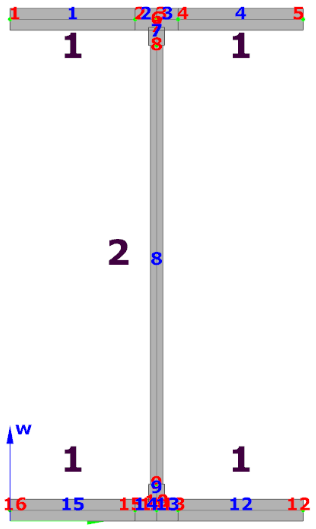


Bending moment value at middle support (4500 mm)

Load combination	**ConSteel first order analysis results** [kNm]	**Reference value**[kNm]	**Difference[%]**
Combination 1	8,07	8,10	0,37

Beam verification

Section classification

ConSteel results	Reference result
	Flange is class 1 Web is class 2

Cross section resistance check

Compression

ConSteel results		Reference results	Difference[%]
Compression			
Utilization	15,4 %		
Section class	2		
Applied part of stand	6.2.4 (1)-(2) - (6.9-6.11) formula		
N_{Ed}	-200,0 kN	No results	
$N_{c,Rd}$	1 297,1 kN		
A	4 716,6 mm²		
f_y	275,0 N/mm²		
γ_{M0}	1,0		

Bending about the minor axis

ConSteel results		Reference results	Difference[%]
Bending about the minc		$M_{c,z,Rd}$=32,7 kN	1,84
Utilization	25,1 %		
Section class	2		
Applied part of stanc	6.2.5 (1)-(3) - (6.12-6.15) formula		
$M_{z,Ed}$	-8,1 kNm		
$M_{z,c,Rd}$	32,1 kNm		
$W_{pl,z,min}$	116 809,6 mm³		
f_y	275,0 N/mm²		
γ_{M0}	1,0		

Major axis shear

ConSteel results		Reference results	Difference[%]
Major axis shear		No results	
Utilization	1,8 %		
Section class	2		
Applied part of stanc	6.2.6 (1)-(3) - (6.17, 6.18) formula		
$V_{y,Ed}$	9,0 kN		
$V_{y,c,Rd}$	506,7 kN		
A_y	3 191,5 mm²		
f_y	275,0 N/mm²		
γ_{M0}	1,0		

Stability check of the beam

Weak axis buckling

ConSteel results		Reference results	Difference[%]
N_{Ed}	200,0 kN		
$N_{b,Rd}$	448,6 kN	$N_{b,rd}$=449kN	0,09
N_{cr}	584,0 kN		
L	4500 mm		
k	0,999		
λ	1,490	λ=1,490	0,0
α	0,340		
Φ	1,830	ϕ=1,829	0,06
χ	0,346	χ=0,346	0,0
A	4716,6 mm²		
f_y	275,0 N/mm²		
γ_{M1}	1,0		

Interaction of buckling and bending

ConSteel results		Reference results	Difference[%]
Summary of results		66,6%	0,01
Used capacity	67,2%		
Used part of standard:	6.3.3 (6.61-6.62) formula		
Applied method for interaction factors	Method 2 (Annex B)		
Weak axis buckling			
Summary of results			
Used capacity in lateral buckling:	44,6%		
Place of the dominant cross section:	0 mm from the first node		
Number of the dominant finite element:	1		
Place of the dominant FE node:	j		
Class of the dominant cross section for compres	2		
Used part of standard:	6.3.1(6.46-6.49) formula		
Detailed calculation			
Results of minor axis bending			
Capacity of minor axis bending check:	25,1%		
Place of the dominant cross section:	4500 mm from the first n		
Number of the dominant finite element:	16		
Place of the dominant FE node:	j		
Class of the dominant cross section for bending:	1		
Used part of standard:	6.2.5 (6.12-6.15) formula		
$M_{z,Ed}$	8,1 kNm		
$M_{z,b,Rd}$	32,1 kNm		
$W_{pl.z}$	116809,6 mm^3		
f_y	275,0 N/mm^2		
γ_{M1}	1,0		
Interaction factors			
Used part of standard:	Annex B, Table B1-B3		
k_{zz}	0,898	k_{yy}=0,893	0,56
C_{mz}	0,553	C_{mz}=0,55	0,55

WE-40 Simply supported beam

Figure 73 simply supported beam. The lateral deflections and twist rotations are prevented at midspan.

A) Verification

The Behaviour and Design of Steel Structure to EC3 (fourth edition): 7.7.4 Example 4

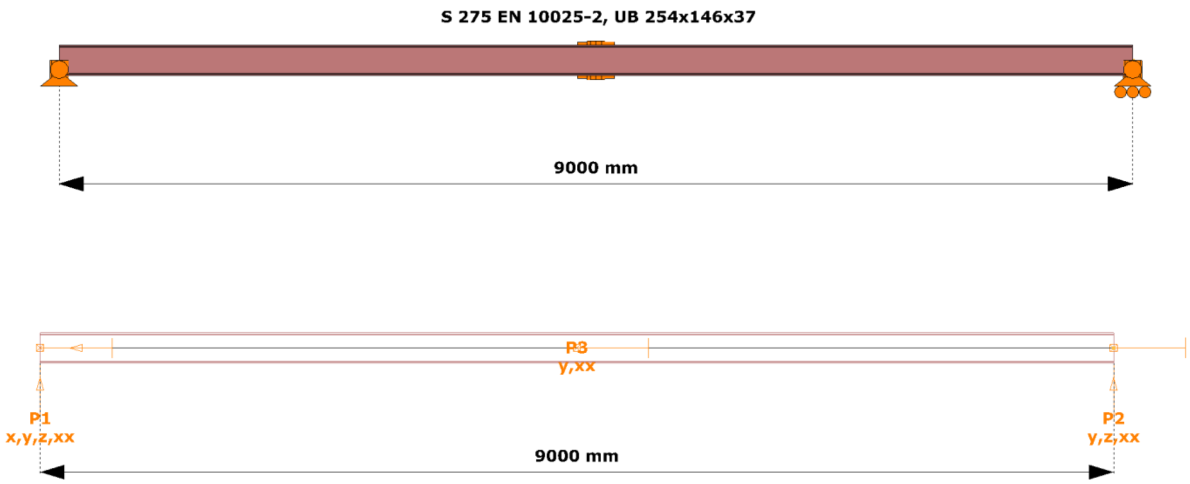
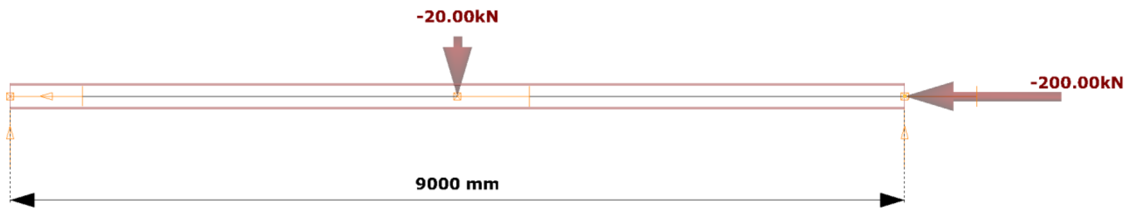


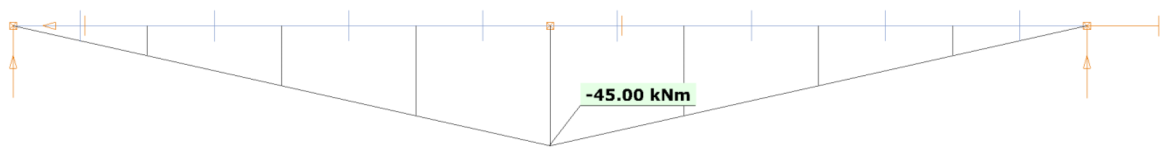
Fig.73 *Simply supported beam*

Loads



Analysis results

Moment diagram

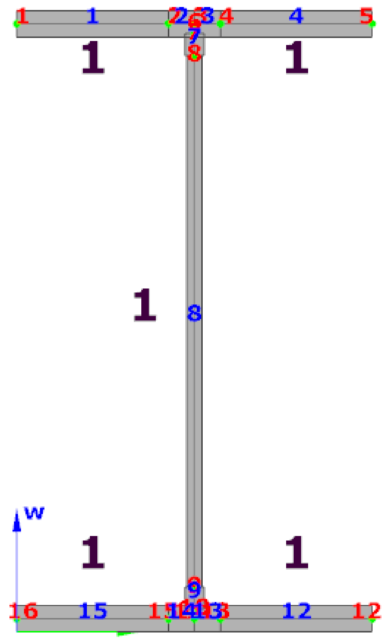


Bending moment value at midspan (4500 mm)

Load combination	ConSteel first order analysis results [kNm]	Reference value[kNm]	Difference[%]
Combination 1	45,00	45,00	0,0

Beam verification

Section classification

ConSteel results	Reference result
	All plates are class 1

Cross section resistance check

Compression

ConSteel results		Reference results	Difference[%]
Compression			
Utilization	15,4 %		
Section class	1		
Applied part of stand	6.2.4 (1)-(2) - (6.9-6.11) formula		
N_{Ed}	-200,0 kN	No results	
$N_{c,Rd}$	1 297,1 kN		
A	4 716,6 mm²		
f_y	275,0 N/mm²		
γ_{M0}	1,0		

Bending about the major axis

ConSteel results		Reference results	Difference[%]
Bending about the majc			
Utilization	33,9 %		
Section class	1		
Applied part of stand	6.2.5 (1)-(3) - (6.12-6.15) formula		
$M_{y,Ed}$	-45,0 kNm	$M_{c,Rd}$=132,8 kN	0,0
$M_{y,c,Rd}$	132,9 kNm		
$W_{pl,y,min}$	483 230,8 mm^3		
f_y	275,0 N/mm^2		
γ_{M0}	1,0		

Minor axis shear

ConSteel results		Reference results	Difference[%]
Minor axis shear			
Utilization	3,6 %		
Section class	1		
Applied part of stand	6.2.6 (1)-(3) - (6.17, 6.18) formula		
$V_{z,Ed}$	-10,0 kN		
$V_{z,c,Rd}$	279,3 kN	No results	
A_z	1 759,4 mm^2		
f_y	275,0 N/mm^2		
γ_{M0}	1,0		

Stability check of the beam

Strong axis buckling

ConSteel results		Reference results	Difference[%]
N_{Ed}	200,0 kN		
$N_{b,Rd}$	902,6 kN	$N_{b,rd}$=900kN	0,29
N_{cr}	1418,0 kN		
L	9000 mm		
k	0,999		
λ	0,956	λ=0,960	0,42
α	0,210		
Φ	1,037	ϕ=1,041	0,38
χ	0,696	χ=0,693	0,43
A	4716,6 mm^2		
f_y	275,0 N/mm^2		
γ_{M1}	1,0		

Weak axis buckling

ConSteel results		Reference results	Difference[%]
N_{Ed}	200,0 kN		
$N_{b,Rd}$	448,6 kN	$N_{b,rd}$=449kN	0,09
N_{cr}	584,0 kN		
L	4500 mm		
k	0,999		
λ	1,490	λ=1,490	0,0
α	0,340		
Φ	1,830	ϕ=1,829	0,06
χ	0,346	χ=0,346	0,0
A	4716,6 mm^2		
f_y	275,0 N/mm^2		
γ_{M1}	1,0		

Lateral-torsional buckling (see 6.15.2 page 278)

ConSteel results		Reference results	Difference[%]
$M_{y,Ed}$	45,0 kNm		
$M_{y,b,Rd}$	123,6 kNm	$M_{b,rd}$=121,4kN	1,81
M_{cr}	205,7 kNm		
L	4500 mm		
k	0,998		
k_w	0,982		
C_1	1,835		
C_2	1,000		
C_3	0,943		
z_g	0 mm		
λ_{LT}	0,804		
α_{LT}	0,340		
Φ	0,811	ϕ_{LT}=0,828	2,05
χ_{LT}	0,815		
$\chi_{LT,mod}$	0,930		
f	0,876	f=0,878	0,23
k_c	0,752		
$W_{pl.y}$	483230,8 mm^3		
f_y	275,0 N/mm^2		
γ_{M1}	1,0		

Interaction of strong axis buckling and lateral-torsional buckling

ConSteel results		Reference results	Difference[%]
Summary of results			
Used capacity	60,4%	61,2%	1,31
Used part of standard:	6.3.3 (6.61-6.62) formul		
Applied method for interaction factors	Method 2 (Annex B)		
Strong axis buckling			
Summary of results			
Used capacity in lateral buckling:	22,2%		
Place of the dominant cross section:	0 mm from the first node		
Number of the dominant finite element:	1		
Place of the dominant FE node:	j		
Class of the dominant cross section for compres	2		
Used part of standard:	6.3.1(6.46-6.49) formula		
Detailed calculation			
Lateral-torsional buckling			
Summary of results			
Used capacity in lateral-torsional buckling:	36,4%		
Place of the dominant cross section:	4500 mm from the first n		
Number of the dominant finite element:	4		
Place of the dominant FE node:	k		
Class of the dominant cross section for compres	1		
Used part of standard:	6.3.2(6.46-6.49) formula		
Detailed calculation			
Interaction factors			
Used part of standard:	Annex B, Table B1-B3		
k_{yy}	1,051	k_{yy}=1,052	0,10
C_{my}	0,900		

Interaction of weak axis buckling and lateral-torsional buckling

ConSteel results		Reference results	Difference[%]
Summary of results			
Used capacity	76,3%	76,9%	0,78
Used part of standard:	6.3.3 (6.61-6.62) formula		
Applied method for interaction factors	Method 2 (Annex B)		
Weak axis buckling			
Summary of results			
Used capacity in lateral buckling:	44,6%		
Place of the dominant cross section:	0 mm from the first node		
Number of the dominant finite element:	1		
Place of the dominant FE node:	j		
Class of the dominant cross section for compres	2		
Used part of standard:	6.3.1(6.46-6.49) formula		
Detailed calculation			
Lateral-torsional buckling			
Summary of results			
Used capacity in lateral-torsional buckling:	36,4%		
Place of the dominant cross section:	4500 mm from the first n		
Number of the dominant finite element:	4		
Place of the dominant FE node:	k		
Class of the dominant cross section for compres	1		
Used part of standard:	6.3.2(6.46-6.49) formula		
Detailed calculation			
Interaction factors			
Used part of standard:	Annex B, Table B1-B3		
k_{zy}	0,873	k_{zy}=0,873	0,0
C_{mLT}	0,600		

3.2 Simple structures

WE-41 Analysis of a single bay portal frame

Figure 74 shows a single bay portal frame model made from hot rolled sections. The column base joint is pinned all other joints are rigid.

A) Verification

Access Steel example (SX029): Elastic analysis of a single bay portal frame

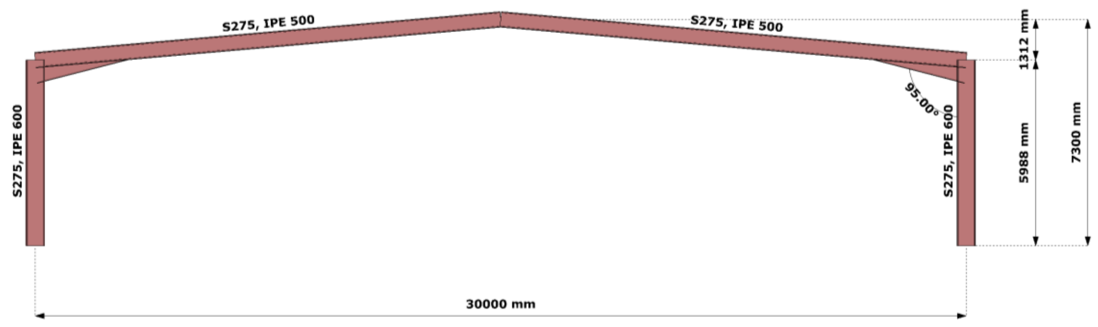


Fig.74 Single bay portal frame with hot-rolled sections

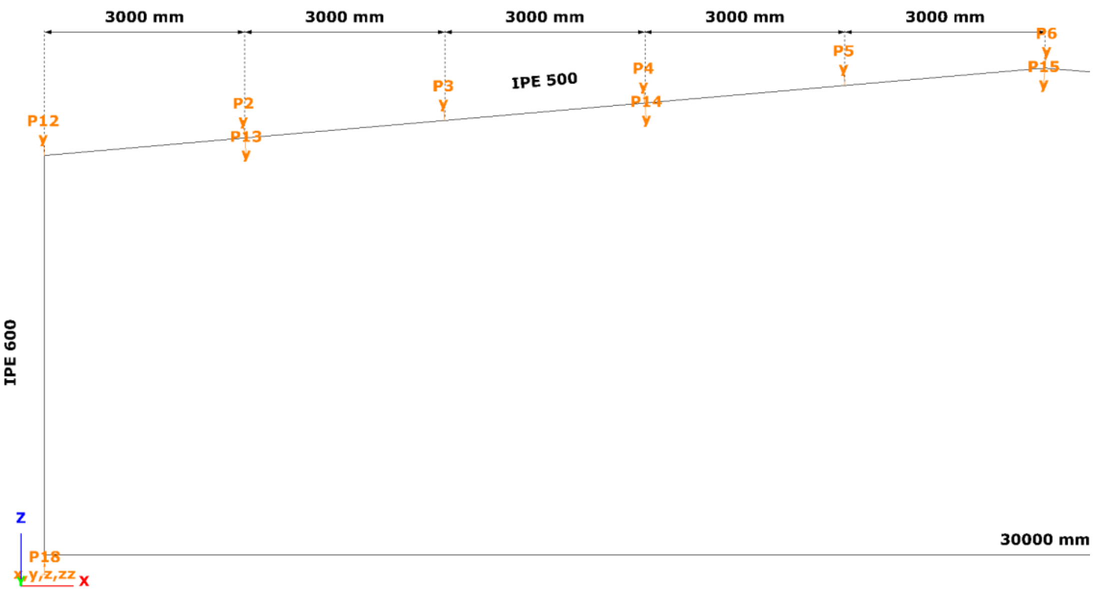
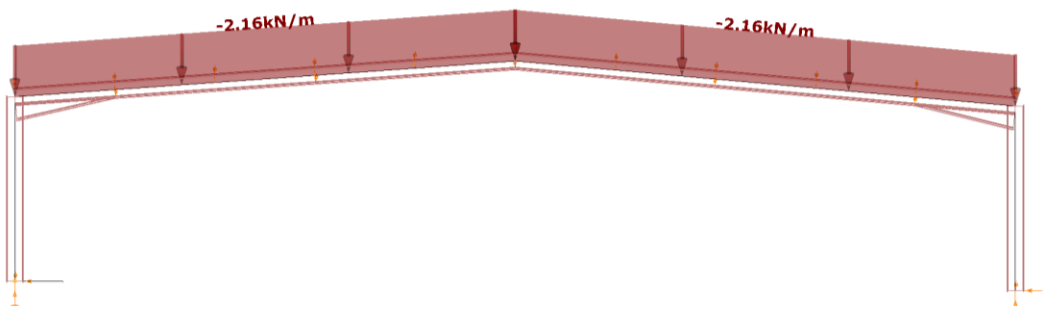


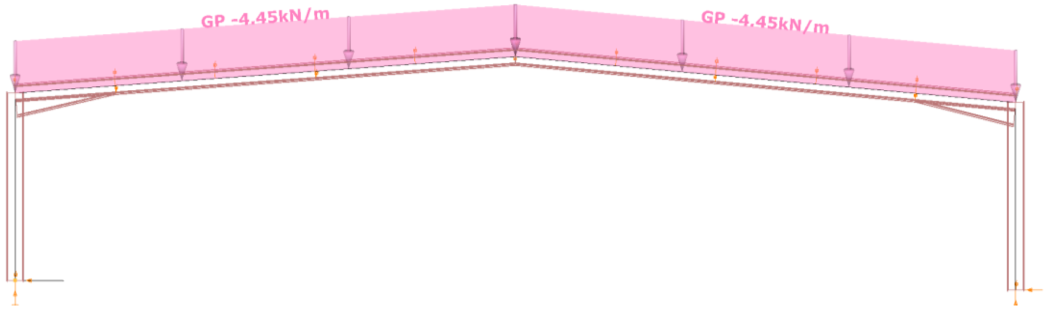
Fig.75 Torsional restraints

Loads

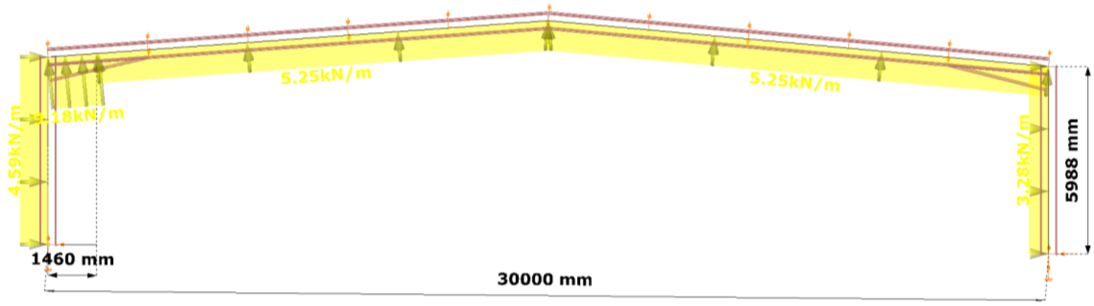
Permanent loads



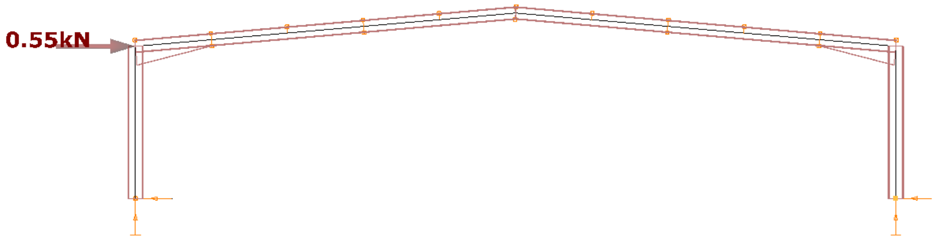
Snow loads



Wind loads



Imperfection load

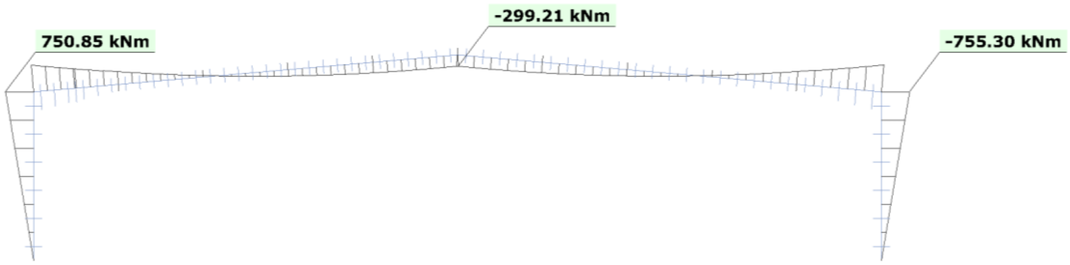


Load combinations

Name	Limit state	Permanent	Imperfection	Snow	Wind left
Load combination-101	(ULS) Ultimate	1,35	1	1,5	0
Load combination-102	(ULS) Ultimate	1	0	0	1,5
Load combination-103	(ULS) Ultimate	1,35	0	1,5	0,9
Load combination-104	(ULS) Ultimate	1	0	1,5	0,9
Load combination-105	(ULS) Ultimate	1,35	0	0,75	1,5
Load combination-106	(ULS) Ultimate	1	0	0,75	1,5

Analysis results

Moment diagram – Combination 101

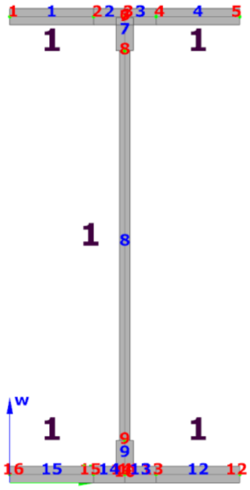


Bending moment value at beam to column joint

Load combination	ConSteel first order analysis results [kNm]		Reference value[kNm]		Difference[%]	
	Left corner	Right corner	Left corner	Right corner	Left corner	Right corner
Combination 101	751	755	748	755	+0,4	0,0
Combination 102	-439	-233	-446	-235	-1,6	-0,9
Combination 103	361	485	356	483	+1,4	+0,4
Combination 104	286	410	281	408	+1,7	+0,4
Combination 105	-132	74	-140	72	-5,7	+2,7
Combination 106	-207	0,6	-215	-3	-3,7	-

Column verification

Section classification

ConSteel results	Reference result
	All plates are class 1

Cross section resistance check

Compression

ConSteel results		Reference results	Difference[%]
Compression		N_{Rd}=4290 kN	0
Utilization	3,8 %		
Section class	1		
Applied part of standard	6.2.4 (1)-(2) - (6.9-6.11) formula		
N_{Ed}	-163,7 kN		
$N_{c,Rd}$	4 289,6 kN		
A	15 598,4 mm^2		
f_y	275,0 N/mm^2		
γ_{M0}	1,0		

Bending about the major axis

ConSteel results		Reference results	Difference[%]
Bending about the major axis		$M_{y,Rd}$=965,8 kN	0,1
Utilization	79,6 %		
Section class	1		
Applied part of standard	6.2.5 (1)-(3) - (6.12-6.15) formula		
$M_{y,Ed}$	-768,4 kNm		
$M_{y,c,Rd}$	965,9 kNm		
$W_{pl,y,min}$	3 512 399,7 mm^3		
f_y	275,0 N/mm^2		
γ_{M0}	1,0		

Minor axis shear

ConSteel results		Reference results	Difference[%]
Minor axis shear			
Utilization	9,6 %		
Section class	1		
Applied part of standard	6.2.6 (1)-(3) - (6.17, 6.18) formula		
$V_{z,Ed}$	-127,8 kN		
$V_{z,c,Rd}$	1 330,3 kN	V_{Rd}=1330 kN	0,0
A_z	8 378,4 mm²		
f_y	275,0 N/mm²		
γ_{M0}	1,0		

Stability check of the column

Strong axis (y-y) flexural buckling

ConSteel results		Reference results	Difference[%]
Used capacity in lateral buckling:	4,1%		
Place of the dominant cross section:	0 mm from the first nc		
Number of the dominant finite element:	1		
Place of the dominant FE node:	j		
Class of the dominant cross section for compress	1		
Used part of standard:	6.3.1(6.46-6.49) form		
N_{Ed}	173,3 kN		
$N_{b,Rd}$	4209,2 kN		
N_{cr}	53258,8 kN	N_{cr}=53190 kN	0,1
L	5988 mm		
k	1,000		
λ	0,284	λ=0,284	0,0
α	0,210		
Φ	0,549		
χ	0,981	χ=0,9813	0,0
A	15598,5 mm²		
f_y	275,0 N/mm²		
γ_{M1}	1,0		

Weak axis (z-z) flexural buckling

ConSteel results		Reference results	Difference[%]
Used capacity in lateral buckling:	11,6%		
Place of the dominant cross section:	0 mm from the first n		
Number of the dominant finite element:	1		
Place of the dominant FE node:	j		
Class of the dominant cross section for compres	1		
Used part of standard:	6.3.1(6.46-6.49) form		
N_{Ed}	173,3 kN		
$N_{b,Rd}$	1493,1 kN		
N_{cr}	1947,1 kN	N_{cr}=1956 kN	0,5
L	5988 mm		
k	1,000		
λ	1,484	λ=1,484	0,0
α	0,340		
Φ	1,820		
χ	0,348	χ=0,3495	0,4
A	15598,5 mm^2		
f_y	275,0 N/mm^2		
γ_{M1}	1,0		

Lateral torsional buckling

ConSteel results		Reference results	Difference[%]
Used capacity in lateral-torsional buckling:	91,0%		
Place of the dominant cross section:	5988 mm from the firs		
Number of the dominant finite element:	6		
Place of the dominant FE node:	k		
Class of the dominant cross section for compres	1		
Used part of standard:	6.3.2(6.46-6.49) form		
$M_{y,Ed}$	753,3 kNm		
$M_{y,b,Rd}$	827,6 kNm		
M_{cr}	1431,1 kNm	M_{cr}=1351 kNm	5,9
L	5988 mm		
k	1,000		
k_w	1,000		
C_1	1,879		
C_2	1,000		
C_3	0,939		
z_g	0 mm		
λ_{LT}	0,822	λ_{LT}=0,8455	2,8
α_{LT}	0,490		
Φ	0,857		
χ_{LT}	0,750	χ_{LT}=0,7352	2,0
$\chi_{LT,mod}$	0,856		
f	0,876		
k_c	0,752		
$W_{pl.y}$	3515482,3 mm^3		
f_y	275,0 N/mm^2		
γ_{M1}	1,0		

Interaction factors

ConSteel results		Reference results	Difference [%]
k_{zy}	0,513	k_{zy}=0,5138	0,2
C_{my}	0,963	C_{my}=0,9641	0,1
C_{mLT}	1,000	C_{mLT}=0,9843	1,6
μ_z	0,940	μ_z=0,9447	0,5
C_{zy}	0,927	C_{zy}=0,9318	0,5
k_{yy}	0,982	k_{yy}=0,9818	0,0
C_{my}	0,963	C_{my}=0,9641	0,2
C_{mLT}	1,000	C_{mLT}=0,9843	1,6
μ_y	1,000	μ_y=0,9999	0,0
C_{yy}	0,984	C_{yy}=0,8739	12,9

WE-42 Analysis of a continuous column in a multi-storey building using an H-section

Figure 77 shows a multi-storey frame model made from hot rolled sections. It is calculated with two different support systems. The designed column is signed with pink colour.

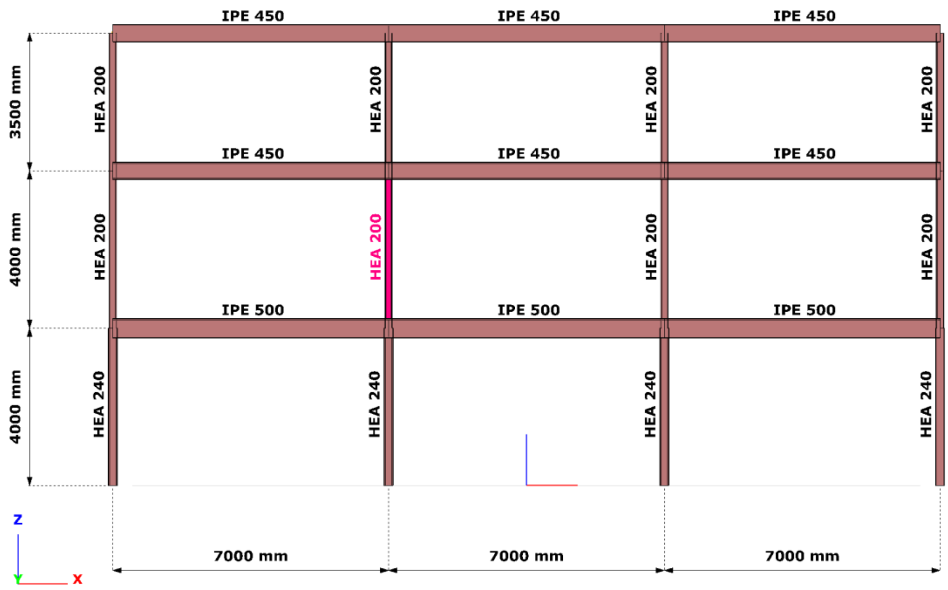


Fig.77 Multi-storey frame

A) Verification

Access Steel example (SX010): [Continuous column in a multi-storey building using an H-section]

Loads

Normal force on the top of the columns: 743 kN

a) non-sway frame

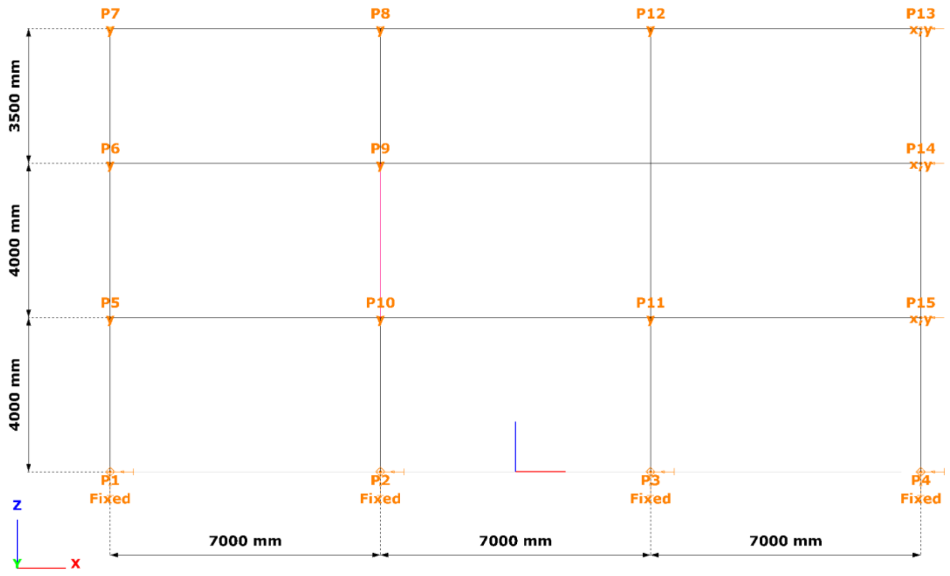


__Fig.78__ Support of the non-sway frame

Effective length factor
According to the Access Steel example: 0,601

Buckling resistance of the column

ConSteel results		Reference results	Difference [%]
N_{Ed}	743,0 kN		
$N_{b,Rd}$	1784,9 kN	Nb,rd=1784kN	+0,05
N_{cr}	13242,8 kN	Ncr=13250kN	-0,05
L	4000 mm		
k	0,601		
λ	0,380	λ=0,38	0
α	0,340		
Φ	0,603	ϕ=0,603	0
χ	0,934	χ=0,934	0
A	5383,2 mm^2		
f_y	355,0 N/mm^2		
γ_{M1}	1,0		

b) sway frame

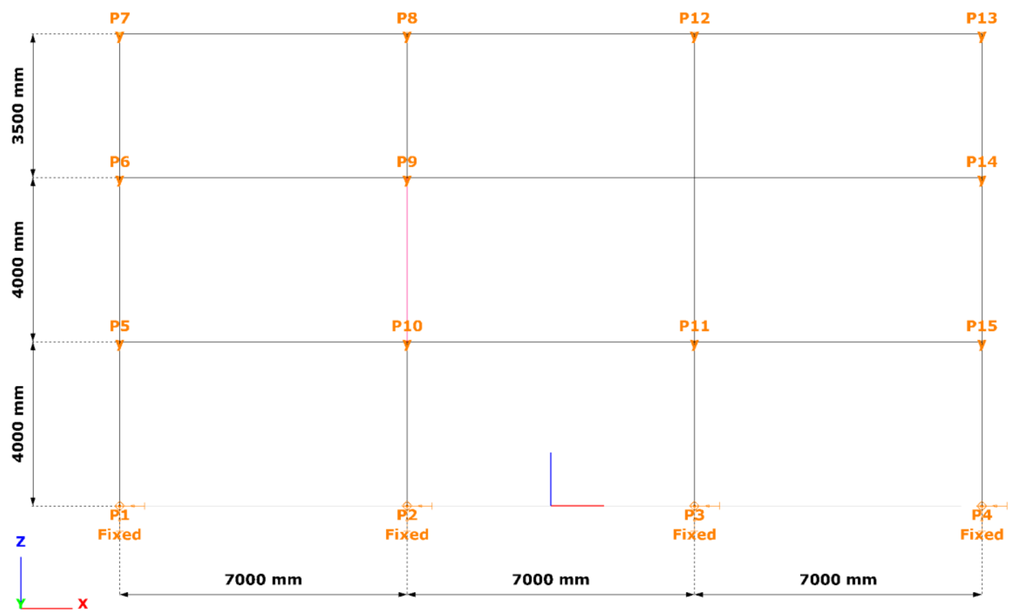


Fig.36 *Support of the sway frame*

Effective length factor
According to the Access Steel example: 1,079

Buckling resistance of the column

ConSteel results		Reference results	Difference [%]
N_{Ed}	743,0 kN		
$N_{b,Rd}$	1516,9 kN	$N_{b,rd}$=1516kN	+0,05
N_{cr}	4108,5 kN	N_{cr}=4102kN	+0,16
L	4000 mm		
k	1,079		
λ	0,682	λ=0,682	0
α	0,340		
Φ	0,815	ϕ=0,815	0
χ	0,794	χ=0,794	0
A	5383,2 mm²		
f_y	355,0 N/mm²		
γ_{M1}	1,0		

4. Special issues

WE-43 Dynamic analysis of a footbridge

Figure 79 shows a 120m span steel footbridge. This example shows the comparison of the dynamic Eigen frequencies with other software products and with the on-site measurements. (The ConSteel model was created by Péter Kolozsi M.Sc structural engineer student at BUTE.)

Fig.79 *Footbridge*

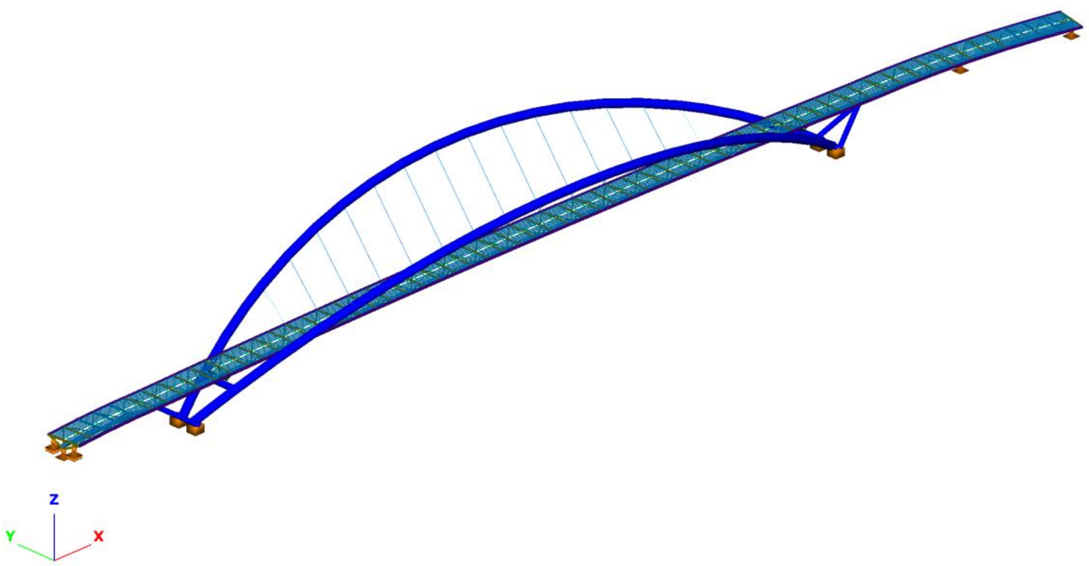


Fig.80 *Footbridge ConSteel model*

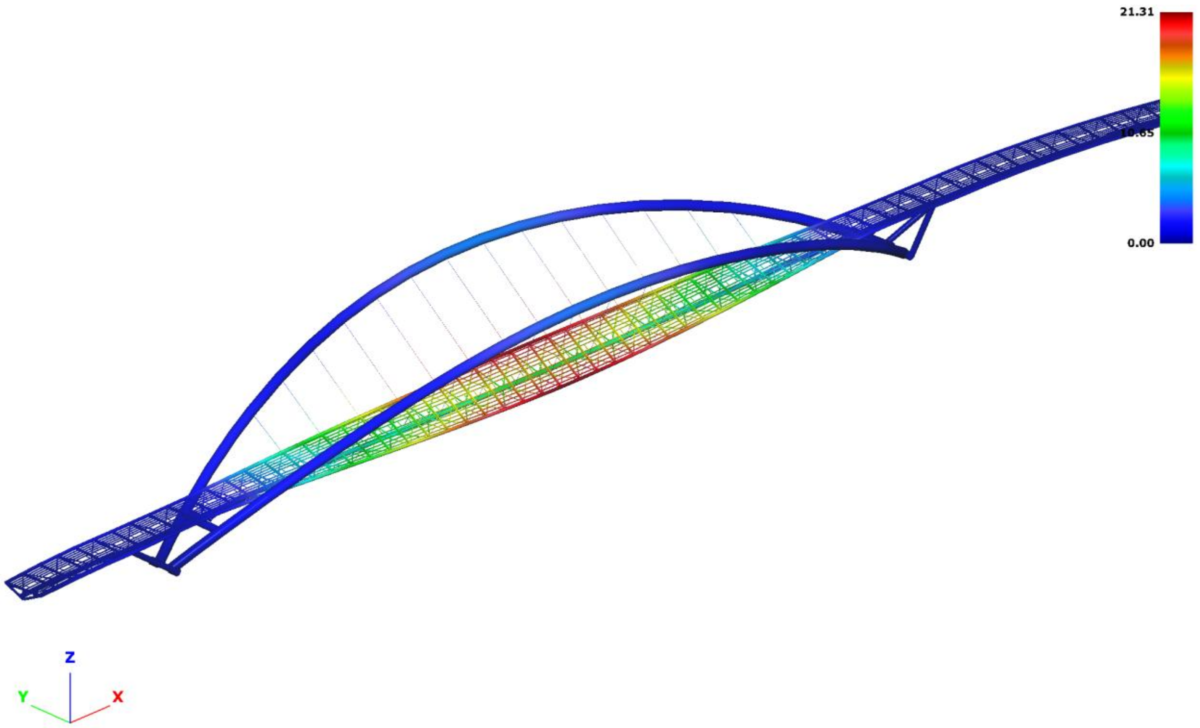


Fig.81 *First dynamic eigenshape (0,57 Hz)*

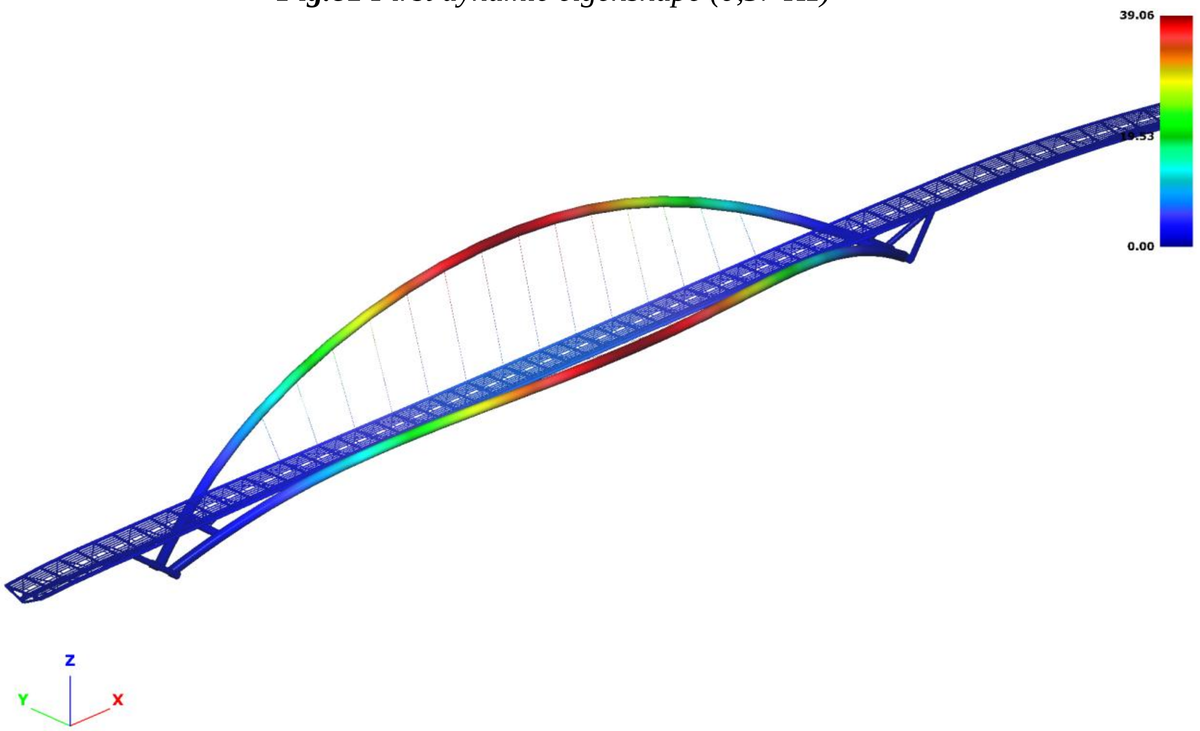


Fig.82 *Second dynamic eigenshape (0,61 Hz)*

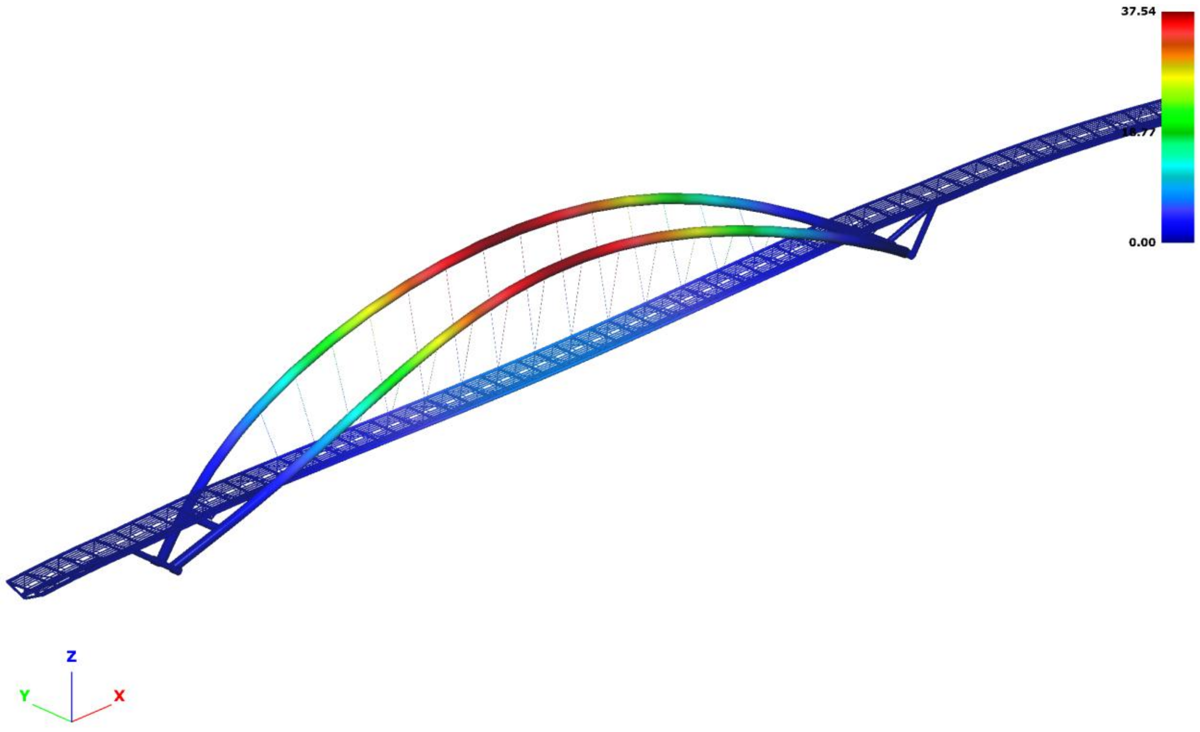


Fig.82 *Third dynamic eigenshape (1,30 Hz)*

	Eigenfrequencies [Hz]									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
ConSteel	0,57	0,61	1,30	1,41	1,51	1,68	2,26	2,41	2,86	2,90
Midas Civil	0,58	0,62	1,24	1,32	1,52	1,68	2,12	2,37	2,86	2,86
Ansys	0,60	0,61	1,15	1,46	1,52	1,74	2,15	2,44	2,83	2,89
Measurements at site #1	0,54	0,56	1,10	1,46	1,46	1,68	2,15	2,54	2,83	2,95
Measurements at site #2	0,71	0,71	1,22	1,49	1,49	1,81	2,31	2,59	2,83	2,95

5. Reference publications with ConSteel results

Feldmann, M.; Sedlacek, G.; Wiescholllek, M.; Szalai, J.: Biege- und Biegedrillknicknachweise nach Eurocode 3 anhand von Berechnungen nach Theorie 2. Ordnung. In: Stahlbau, 1 (2012), S. 1-12 (PDF)

Wiescholllek, M.; Schillo, N.; Feldmann, M.; Sedlacek, G.: Lateral-torsional buckling checks of steel frames using second-order analysis. In: Steel Construction - Design and Research, 2 (2012), S. 71-86

Wiescholllek, M.; Feldmann, M.; Szalai, J.; Sedlacek, G.: Biege- und Biegedrillknicknachweise nach Eurocode 3 anhand von Berechnungen nach Theorie 2. Ordnung. In: Festschrift Gerhard Hanswille, Institut für Konstruktiven Ingenieurbau, Bergische Universität Wuppertal (2011), S. 73-95

Szalai, J.: The "General Method"of EN 1993-1-1 New Steel Constructions April 2011 (PDF)

Szalai, J.: Practical application of the "General Method" of EN 1993-1-1 New Steel Constructions May 2011 (PDF)

Z. Nagy and M. Cristutiu: Local and Global Stability Analysis of a Large Free Span Steel Roof Structure Civil-Comp Press, 2012 Proceedings of the Eleventh International Conference on Computational Structures Technology

Z. Nagy and M. Cristutiu: Application of monitoring to ensure structural robustness 6th European Conference on Steel and Composite Structures. Edited by Dunai L at al. Budapest, Hungary, 2011.

Szalai J, Papp F. Nowe trendy w normach: EUROKOD 3 – efektywne globalne projektowanie konstrukcji. Inzynier Budownictwa, 81/2, pp. 39-43. 2011.

Szalai J, Papp F. Nowe trendy w normach: EUROKOD 3 – efektywne globalne projektowanie konstrukcyjne Analiza oparta na modelu 3D przy użyciu ogólnej metody elementów skończonych belkowo-słupowych. Inzynier Budownictwa, 84/5, pp. 35-42. 2011.

Szalai J, Papp F. Theory and application of the general method of Eurocode 3 Part 1-1. 6th European Conference on Steel and Composite Structures. Edited by Dunai L at al. Budapest, Hungary, 2011.

Wald F, Papp F, Szalai J, Vídenský J. Obecná metoda pro vzpěr a klopení. SOFTWAROVÁ PODPORA NÁVRHU OCELOVÝCH A DŘEVĚNÝCH KONSTRUKCÍ (Software Solutions for Steel and Timber Structures), pp. 48-57., Prague, 2010.

Papp F, Szalai J. New approaches in Eurocode 3 – efficient global structural design. Part 0: An explanatory introduction. Terästiedote (Finnish Steel Bulletin), 5, Helsinki, 2010.

Papp F, Szalai J. New approaches in Eurocode 3 – efficient global structural design. Part 1: 3D model based analysis using general beam-column FEM. Terästiedote (Finnish Steel Bulletin), 5, 2010.

Szalai J. Use of eigenvalue analysis for different levels of stability design. International Colloquium on the Stability and Ductility of Steel Structures. Edited by Batista E at al. Rio de Janeiro, Brasil, 2010.

Badari B, Papp F. Calibration of the Ayrton-Perry resistance formula – A new design formula for LTB of simple beams. 6th European Conference on Steel and Composite Structures. Edited by Dunai L at al. Budapest, Hungary, 2011.