# THEORY AND APPLICATION OF THE GENERAL METHOD OF EUROCODE 3 PART 1-1

Ferenc Papp [a], József Szalai [b]

[a] Budapest University of Technology and Economics, Dept. of Structural Engineering, Hungary
[b] ConSteel Solutions Ltd., Hungary

## INTRODUCTION

The EN 1993 Part 1-1 (EC3-1-1) has introduced a new approach (called the "General Method") to perform lateral-torsional buckling (LTB) assessment of beam-column structural components on the basis of elastic stability analysis. In the last years great research investigations went into the development of the method, see for instance [11,12] and also into the improvement of appropriate design software that is suitable to include the method and applicable for practical solutions [10]. The general objective of this paper is to review this issue from the point of view of the practice and contribute more effectively to understanding and resolving issues in the fields of practical application of the General Method. It is essentially significant to define the minimal analysis tools for the practice which are required for the accuracy of the method but on the other hand simple enough to make the modeling and calculation efficient. The paper briefly presents the theoretical background and the practical application of the elastic stability analysis of beam-columns that is necessary for the accurate evaluation of the General Method. The elastic stability analysis is verified by benchmark examples and also by shell finite element analysis. The application of the design method is demonstrated in the field of irregular structural members, especially web-tapered members and frames. The paper analyses the new theoretical results in the field of LTB of web-tapered members that have led to prohibitive statements in some National Annex for EC3-1-1 concerning the segment method in the analysis of these members. It is shown that a comprehensive design method that is based on an appropriate segmented model and the General Method is efficient as well as reliable for conceptual design and with some restrictions also for detailed design.

## 1 GENERAL METHOD BY EC3-1-1

According to the EC3-1-1 the global stability resistance of regular or irregular structural members or structures composed of these members may be estimated by the following design equation (providing that the effect of the design moment about minor axis may be neglected), see [1]:

$$\frac{N_{Ed}}{\chi_z \cdot A \cdot f_{yd}} + \frac{M_{y.Ed}}{\chi_{LT} \cdot W_y \cdot f_{yd}} \le 1 \qquad (1)$$

where $N_{Ed}$ is the design compressive force, $M_{y.Ed}$ is the design bending moment, $A$ and $W_y$ are the cross-sectional properties, $f_{yd}=f_y/\gamma_{M1}$ is the design strength and $\chi$ is the appropriate reduction factor. *Table 1* shows the design parameters of the design formulas that are related to the basic buckling modes such as flexural buckling, LTB and interaction of previous ones. The $\alpha_{cr}$ elastic critical load amplifier has an essential role in the design equation *Eq.(1)*, which may be computed using global elastic stability analysis.

## 2 GLOBAL ELASTIC STABILITY ANALYSIS

The $N_{cr}$ and $M_{cr}$ elastic critical forces of regular structural members may be calculated by well known equations published in many papers and text books, for example see [2]. For the case of uniform compressive force and bending moment the interaction of flexural buckling and LTB may be estimated by the known formula (see [3]):

*Table 1.* Analogy between the parameters of the buckling formulas

| design parameters | Flexural Buckling formula | LTB formula | **General Method** *Eq.(1)* |
|---|---|---|---|
| slenderness | $\bar{\lambda} = \sqrt{\frac{A \cdot f_y}{N_{cr}}}$ | $\bar{\lambda}_{LT} = \sqrt{\frac{W_y \cdot f_y}{M_{cr}}}$ | $\bar{\lambda}_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr}}}$ |
| elastic critical forces | $N_{cr.y}; N_{cr.z}$ | $M_{cr}$ | - |
| elastic critical load amplifier | - | - | $\alpha_{cr}$ |
| characteristic resistance | $A \cdot f_y$ | $W_y \cdot f_y$ | $\alpha_{ult,k}$ |
| reduction factors | $\bar{\lambda} \rightarrow \chi_y, \chi_z$ | $\chi_{LT}$ | $\bar{\lambda}_{op} \rightarrow \chi_z, \chi_{LT}$ |

$$\left(\frac{M_y}{M_{cr}}\right)^2 = \left(1 - \frac{N}{N_{cr.z}}\right) \cdot \left(1 - \frac{N}{N_{cr.w}}\right) \tag{2}$$

where $N_{cr.w}$ is the elastic critical load for pure torsional buckling mode. For some irregular members the critical forces were derived using basic mechanical principals. For example Mohri at. al. examined the elastic critical moment of mono-symmetric I beam using theoretical investigation and numerical method (Abaqus software), see [4]. The considerable differences in the results were explained by the Wagner effect which was taken into consideration in the theoretical investigation but in the numerical method. The origin of their mistake was routed in the theoretical fact, that in the general beam-column finite element method the transverse load (as well as the shear force) is basically considered in the shear centre. Assuming this rule their "new" theoretical result and the numerical solution would give the same result. Andrade and Camotim examined elastic web-tapered beam using Rayleight-Ritz method taken the pre-buckling effect into consideration, see [5] and [6]. Their new results can be accurately reproduced by the known general thin-walled beam-column finite element method described in the next Section 3.

## 3 GENERAL BEAM-COLUMN FINITE ELEMENT ANALYSIS

Rajasekaran derived the matrix equilibrium equation of the general thin-walled beam-column finite element in explicit form, see [7]:

$$\left(\underline{\underline{K}}_s + \underline{\underline{K}}_g\right) \times \underline{u} = \underline{f} \tag{3}$$

where $\underline{\underline{K}}_s$ is the flexural and $\underline{\underline{K}}_g$ is the geometric stiffness matrices. The stiffness matrices may be derived from the equilibrium of the virtual works:

$$\int_V (\sigma \cdot \delta\varepsilon + \tau_{vu} \cdot \delta\gamma_{vu} + \tau_{wu} \cdot \delta\gamma_{wu}) \cdot dV = \sum_n f_n \cdot \delta u_n \tag{4}$$

In *Eq.(4)* the left hand side expresses the work of the internal stresses on the appropriate virtual strains, while the right hand side expresses the work of the stress resultants on the appropriate virtual displacements. Furthermore, $\sigma$ is the normal stress and $\delta\varepsilon$ is the corresponding virtual normal strain, $\tau$ is the shear stress and $\gamma$ is the corresponding virtual shear strain at the arbitrary point on the counter of the thin-walled cross-section. The $n$ at right hand side denotes the degrees of freedom of the element (*n=14*). The flexural stiffness matrix is expressed in terms of the geometrical properties of the element, while the geometric matrix is expressed in terms of the actual internal forces such as normal force, shear forces and bending moments. Furthermore, the geometric stiffness matrix depends on the Wagner coefficient which can be generally written as

$$\overline{K} = \int_s a^2 \cdot \sigma \cdot t \cdot ds \tag{5}$$

where $a$ is the distance of the counter point of the cross-section to the shear centre, $t$ is the constant wall-thickness. For the FE model of a regular structural member the compatibility condition of warping in any node may be satisfied by the following condition:

$$\sum_{i=1,2} B_i = 0 \tag{6}$$

where $B$ denotes the bimoment in the node, $i$ denotes the finite elements related to the node. The accuracy of the element was examined in general by many of papers, such as [8] and [9]. The elastic stability analysis (solving generalized eigenvalue problem) which is based on *Eq.(3)* provides the $\alpha_{cr}$ elastic critical load amplifier, which is the most important design parameter of the General Method.

## 4 ELASTIC STABILITY ANALYSIS BY FEM

*Figs.1-4* shows some published benchmark models that were analyzed by the general beam-column finite element summarized in Section 3 implemented in the ConSteel software [10]. The numerical method in all cases has given exactly the same results as given by the theory.

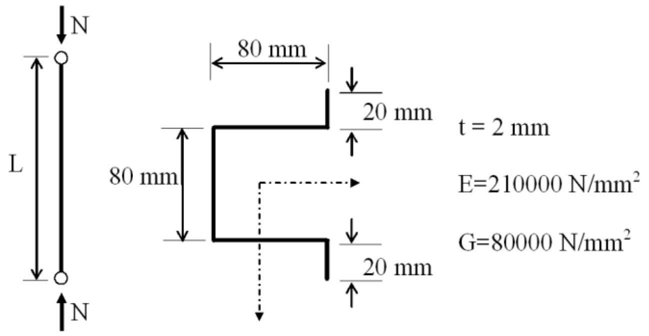


*Fig. 1.* Flexural-torsional buckling by Trahair [3]

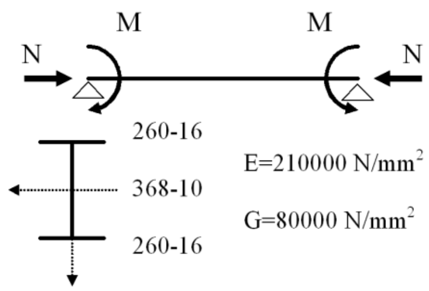


*Fig. 2.* Interaction buckling by theory [3]

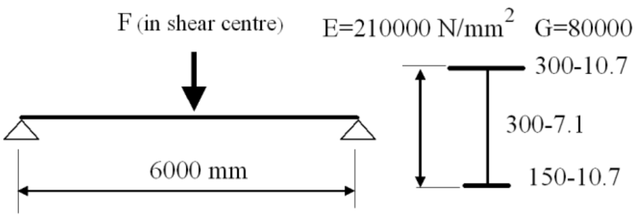


*Fig. 3.* LTB of beams by Mohri [4]

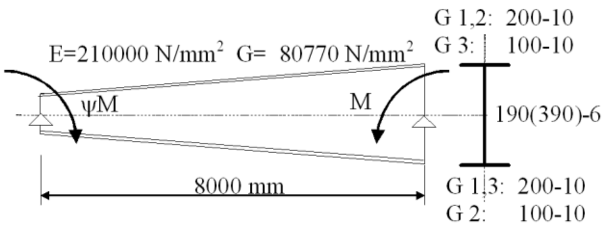


*Fig. 4.* LTB of tapered beams by Andrade [5]

Andrade and Camotim examined the elastic critical load of the tapered cantilever shown in *Fig.5* using analytical and numerical methods, see [6]. The results are summarized in *Table 2* including our examinations using the described general beam-column FEM (beam7) and triangular shell FEM (shell3). It can be seen that the greatest difference between the shell analysis is less than 3,5%. The differences between the shell and the general beam-column analysis are also close to each other, except the model of L=4.0 meters with top flange load where it is less than 7%.

## 5 PARAMETRIC STUDY ON MEMBER RESISTANCE

*Fig.6* shows the model of a web-tapered structural member. The higher end of the member is subjected to $M=M_{y.el.Rd}/2$ bending moment and $N=200$ kN compressive force.

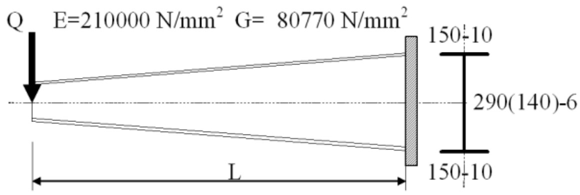


*Fig. 5.* LTB of cantilever

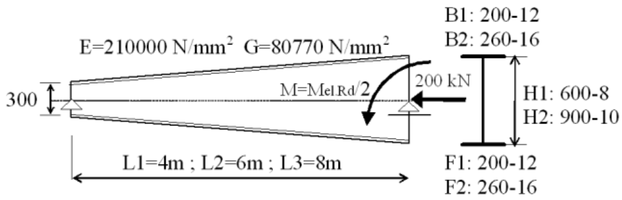


*Fig. 6.* Member model for parametric study

*Table 2.* Critical transverse load for tapered cantilever (see *Fig.5.*)

| L [m] | Qcr [kN] | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | top flange | | | centroid | | | bottom flange | | |
| | FEA [12] | ConSteel | | FEA [12] | ConSteel | | FEA [12] | ConSteel | |
| | | beam7 | shell3 | | beam7 | shell3 | | beam7 | shell3 |
| 4 | 31.6 | 33.8 | 31.5 | 53.5 | 55.7 | 55.3 | 70.0 | 70.2 | 69.9 |
| 6 | 15.2 | 15.4 | 14.7 | 20.3 | 20.2 | 19.8 | 23.7 | 23.5 | 22.9 |
| 8 | 8.4 | 8.3 | 8.2 | 10.2 | 10.1 | 10.0 | 11.4 | 11.4 | 11.3 |

In the parametric study we examined the cases that were given by 4, 6, and 8 meter lengths, 200-12 and 260-16 flanges and 600 and 800 mm web heights. *Table 3* contains the eight realistic cases for which the $\alpha_{cr}$ elastic critical load amplifier was computed by two numerical methods: (i) general beam-column FEM with *n=16* uniform segments (beam7), see *Fig.7*; (ii) triangular shell FEM with 25 mm size elements (shell3), see *Fig.8*. The table contains the most important design parameters of the General Method. The last column shows the utilization of the global stability resistance. The greatest difference between the two FEMs was given by the model of L=8 meters with 260-16 flanges and 900 mm web height:

- difference in elastic critical load amplifiers : $\dfrac{\alpha_{cr}^{beam7}}{\alpha_{cr}^{shell3}} = 0{,}932$
- difference in utilization of global stability: $\dfrac{\eta^{beam7}}{\eta^{shell3}} = 1{,}042$

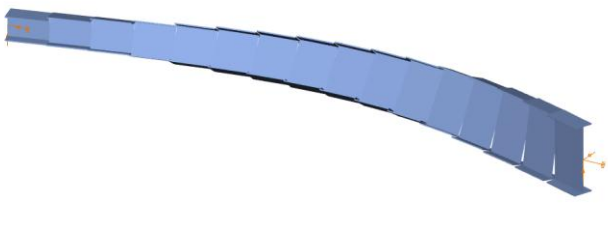

*Fig. 7.* LTB by segmented FEA (beam7)

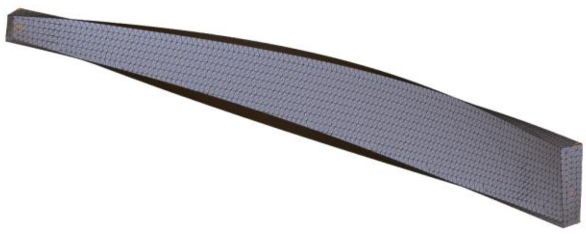

*Fig. 8.* LTB by shell FEA (shell3)

## 6 STUDY ON TAPERED FRAME

*Fig.9* shows a pin supported frame which is composed of web-tapered structural members. The model is loaded by 10 kN/m vertical uniformly distributed load. The span of the frame is 26,0 meters, the flanges are 300-16. The web-height (measured by the distance of the flange centres) of the members is changing from 300 mm to 1000 mm. In the beam-column joints the beam flanges run over as web stiffeners of the columns. The model is supported laterally in the beam-to-beam and beam-to-column joints and in the half of the columns as well as in the quarter nodes of the beams. *Table 3* contains the maximum vertical deflections and the $\alpha_{cr}$ elastic critical load amplifiers

*Table 3.* Parametric study on member resistance (see *Fig.6*)

| model | | | FEM | $\alpha_{cr}$ | $\alpha_{ult,k}$ | $\lambda_{op}$ | $\chi_z$ | $\chi_{LT}$ | $\eta$ |
|---|---|---|---|---|---|---|---|---|---|
| L [mm] | B-tf [mm] | H-tw [mm] | | | | | | | |
| 4000 | 200-12 | 600-8 | beam7 | 3.26 | 1.668 | 0.715 | 0.715 | 0.633 | 0.931 |
| | | | shell3 | 3.32 | | 0.709 | 0.719 | 0.637 | 0.925 |
| 6000 | | | beam7 | 1.60 | | 1.021 | 0.528 | 0.457 | 1.287 |
| | | | shell3 | 1.61 | | 1.018 | 0.530 | 0.458 | 1.282 |
| 4000 | | 900-10 | beam7 | 2.49 | 1.758 | 0.840 | 0.637 | 0.555 | 1.010 |
| | | | shell3 | 2.60 | | 0.822 | 0.648 | 0.566 | 0.991 |
| 6000 | | | beam7 | 1.21 | | 1.205 | 0.431 | 0.374 | 1.499 |
| | | | shell3 | 1.29 | | 1.167 | 0.450 | 0.390 | 1.439 |
| 6000 | 260-16 | 600-8 | beam7 | 3.23 | 1.728 | 0.731 | 0.705 | 0.623 | 0.917 |
| | | | shell3 | 3.30 | | 0.724 | 0.710 | 0.628 | 0.910 |
| 8000 | | | beam7 | 2.02 | | 0.925 | 0.585 | 0.507 | 1.124 |
| | | | shell3 | 2.07 | | 0.914 | 0.591 | 0.513 | 1.111 |
| 6000 | | 900-10 | beam7 | 2.46 | 1.791 | 0.850 | 0.629 | 0.548 | 1.008 |
| | | | shell3 | 2.63 | | 0.825 | 0.646 | 0.564 | 0.978 |
| 8000 | | | beam7 | 1.51 | | 1.089 | 0.490 | 0.424 | 1.301 |
| | | | shell3 | 1.62 | | 1.051 | 0.551 | 0.442 | 1.249 |

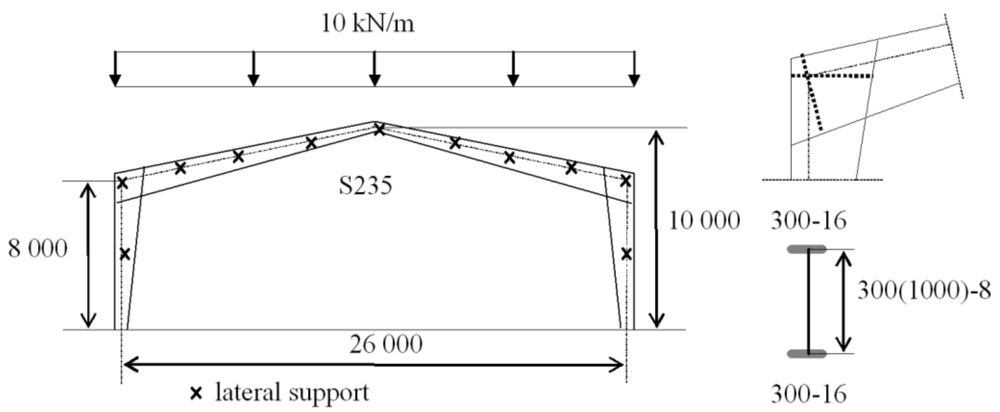


*Fig. 9.* Web-tapered frame model

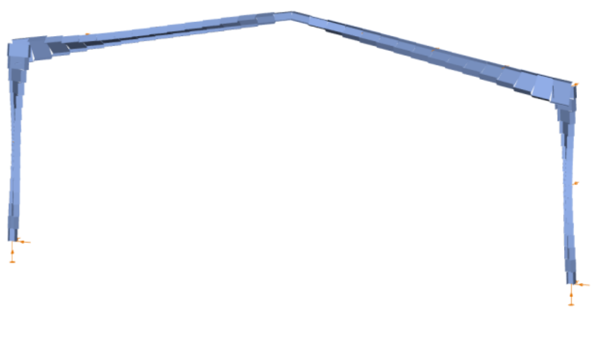

*Fig. 10.* Buckling mode by FEM (beam7)

*Fig. 11.* Buckling mode by FEM (shell3)

computed by two numerical methods: (i) general beam-column FEM with $n=16$ uniform segments (beam7), see *Fig.10*; (ii) triangular shell FEM with 50 mm size elements (shell3), see *Fig.11*. The difference between the load amplifiers is less than 2%. The example illustrates the accuracy of the segment method which is based on the uniform general beam-column finite element.

*Table 4.* Study on tapered frame (see *Fig.9*)

| FEA model | maximum deflection [mm] | critical load amplifier ($\alpha_{cr}$) |
|---|---|---|
| „beam7” | 103 | 1.61 |
| „shell3” | 93.4 | 1.58 |

## 7 SUMMARY AND ACKNOWLEDGMENT

The paper presented the practical application of the General Method which has been introduced by the EC3-1-1 primarily for the stability design of structural components having some geometrical, loading or supporting irregularity. The method is reviewed by the evaluation of web tapered members and frames. It was shown that for realistic geometry and loading excluding the influence of local and distortional buckling the application of segmented general beam-column finite element model is suitable for practical applications. The result components of the structural design – elastic critical values, slenderness, reduction factor and the final resistance utilization – were compared to values calculated by shell finite element model and the deviations were was found to be insignificant. On the other hand the use of shell finite element model makes the method very inefficient due to the large modeling costs while the segment model keeps the expected simplicity that can make the application of General Method attractive for the practice.

## REFERENCES


[1] Eurocode 3: Design of steel structures – Part 1-1, “General rules and rules for buildings”, 2005

[2] Trahair, N.S., Bradford, M.A., Nethercot, D.A. and Gardner, L., “The behaviour and design of steel structures to EC3”, Taylor & Francis New York, 2008

[3] Trahair, N.S., “Flexural-torsional buckling of structures”, E & FN Spon London (1993), p. 99

[4] Mohri, F., Brouki, A. and Toth, J.C., “Theoretical and numerical stability analyses of unrestrained, mono-symmetric thin-walled beams”, Journal of Constructional Steel Research, 59 (2003) 63-90

[5] Andrade, A., Camotim, D., “Lateral-torsional stability behaviour of arbitrary singly symmetric tapered steel beams: a variational formulation”, International Conference on Steel Constructions, Eurosteel, Coimbra 2002,, Conference Proceedings, pp. 107-118.

[6] Andrade, A., Camotim, D. and Borges Dinid, P., “Lateral-torsional buckling of singly symmetric web-tapered thin-walled I-beams: 1D model vs. shell FEA”, Computer & Structures 85 (2007) 1343-1359

[7] Chen, W.F. and Atsuta, T., “Theory of Beam-Columns: Space behavior and design”, Vol. 2, McGraw-Hill, 1977, pp. 539-564

[8] Lip H. The, “Beam element verification for 3D elastic steel frame analysis”, Computers and Structures 82 (2004) 1167-1179

[9] Turkalj, G., Brnic, J. and Prpic-Orsic, J., “Large rotation analysis of elastic thin-walled beam-type structures using ESA approach”, Computers & Structures 81 (2003) 1851-1864

[10] www.consteel.hu

[11] Szalai, J., Papp, F., “On the theoretical background of the generalization of Ayrton–Perry type resistance formulas”, Journal of Constructional Steel Research 66 (2010) 670-679.

[12] Bijlaard, F., Feldmann, M., Naumes, J., Sedlacek, G., “The “general method” for assessing the out-of-plane stability of structural members and frames and the comparison with alternative rules in EN 1993 – Eurocode 3 – Part 1-1” Steel Construction 3 (2010) 19-33.