

European and United States approaches for steel storage pallet rack design. Part 2: Practical applications

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ABSTRACT

A two-part paper has been written to summarise the main results of a comparative study on the design provisions currently adopted in Europe (EU) and the United States (US) for steel storage pallet racks. In part 1 (*Discussion and general comparisons*), key features of the verification procedures for thin-walled cold-formed members as well as of the design alternatives permitted by the EU and US rack codes have been discussed, pointing out the most relevant similarities and differences. The present *part 2* applies six design alternatives to medium-rise pallet racks unbraced in the longitudinal direction. In particular, the proposed research outcomes are based on the design of 216 racks differing for configurations, geometry of components and degree of rotational stiffness of beam-to-column joints and base-plate connections. Results are presented and compared directly to each other in term of safety index in order to allow for a concrete appraisal of the most relevant differences between the considered design methods, highlighting also the influence associated with the approaches to modelling the geometric imperfection effects. Finally, Appendix A presents a complete design example to be used as benchmark for researchers and designers, where all the discussed design options are applied.

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1. Introduction

This paper is the second part of a two-part paper focused on the approaches currently admitted for the design of steel storage pallet racks according to the European (EU) [1] and United States (US) [2] provisions. In part 1 (*Discussion and general comparisons*, [3]), key features of both EU and US design codes have been briefly introduced and discussed, mainly with reference to the evaluation of the effective geometric properties of thin-walled members and to the verification checks associated with isolated columns and beam-columns. Furthermore, similarities and differences related to the permitted design procedures have been highlighted: in particular, the direct analysis (EU-DAM), the rigorous analysis (EU-RAM), the general (EU-GEM), and the improved rigorous analysis (EU-IRAM) methods have been described for what concerns the European alternatives. As to the US approaches [4], both the notional load (US-NOLM) and the effective length method (US-ELM) have been introduced. All these six methods have been applied in this second paper to cases of practical interest for routine design. In particular, a parametric study on 216 medium-rise pallet racks has been carried out by varying the number of load levels, the member geometry and the degree of the rotational stiffness of beam-to-column joints and base-plate connections. Furthermore, Appendix A presents a complete benchmark example, where all the design paths admitted by both codes are applied and compared. Generally, uprights are comprised of open cross-sections but also boxed thin-walled members are available on the market for industrial storage systems, which are currently employed not only for pallet racks but also for shelving, drive-in and drive thru racks and warehouses (i.e. cladding racks). The influence of warping torsion, which significantly affects the behaviour and the design of racks composed by mono-symmetric cross-section uprights [5,6], has been herein neglected: it has been decided to make reference to bi-symmetric cross-section uprights, being the core of the present research the investigation of the performances associated with the design alternatives admitted by the codes. Nevertheless, it should be noted that this choice, due to the need of reducing the number of variables affecting research outcomes, does not limit the validity of the conclusions. It is worth mentioning that attention has been focused on the design of uprights and the other key components have been neglected, from the design point of view, in the present study. On the basis of the authors' expertise, the differences associated with the choice of the method of analysis as well as with the verification check

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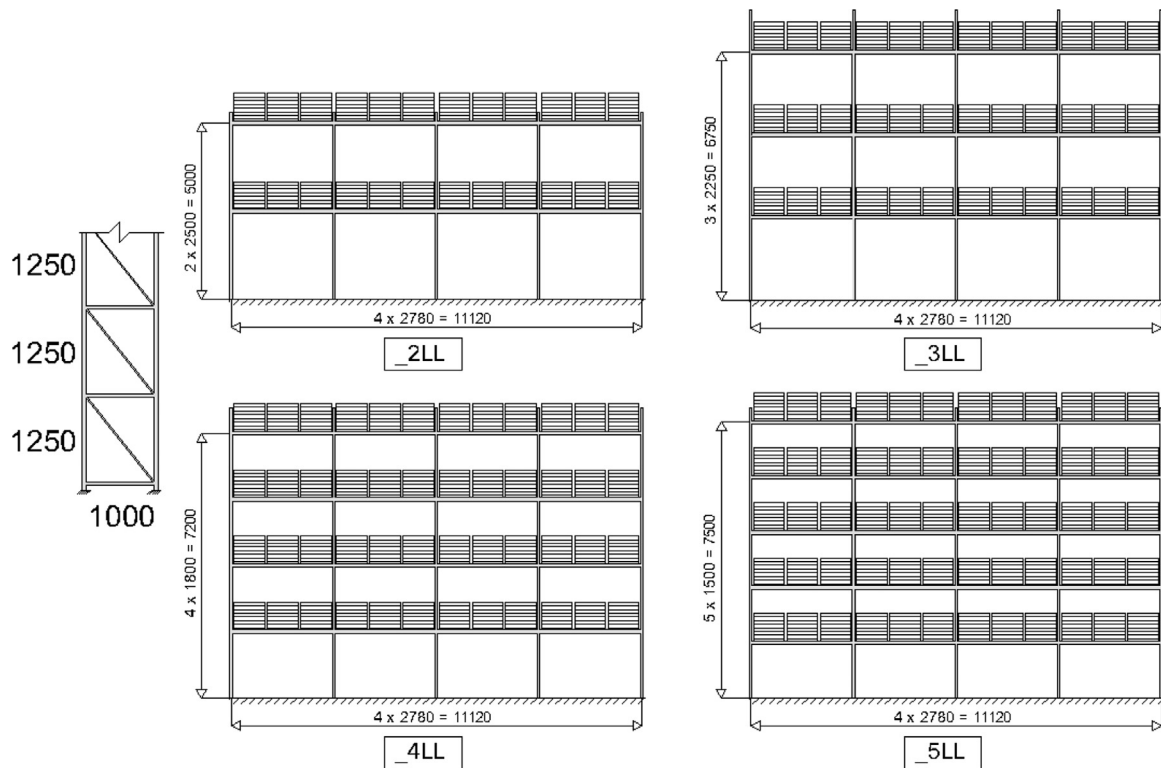


Fig. 1. The considered pallet racks (dimension in millimetres).



Fig. 2. The considered upright cross-sections (dimension in millimetres).

procedures reflect mainly on the design of these vertical components, which are subjected to bending moments and axial force. More limited differences are expected for pallet beams and lacing, which are, for this reason, outside the scope of the present study.

2. The numerical cases

A parametric analysis has been developed for medium-rise double-entry racks, unbraced in the down-aisle direction with six equal bays (Fig. 1) of 2.78 m length: their depth is 1 m and upright frames present Z-panels guaranteeing stability to cross-aisle loads. Three upright cross-sections (identified as M_u, D_u and T_u types) have been considered (Fig. 2), which are represented by a bi-symmetric hollow rectangular cross-section, the closure of which is obtained by overlapping and clamping together the lateral edges of the strip coil. All these uprights belong to class 4 of Eurocode 3 [7] or, equivalently, can be classified as slender members according to AISC provisions [8], i.e. their behaviour is affected by local buckling phenomena. It is worth mentioning that, owing to the impossibility to predict theoretically the effective geometric parameters of these cross-section components, design assisted by testing [9] is required because of the presence of the overlapping zone, internal stiffeners and two connection points on each cross-section side to quickly connect the pallet beams (Fig. 3). With reference to the gross cross-section, the value of the area, second moment of area and section moduli are reported in Table 1, together with the uniform and warping torsion constants. Furthermore, in the same table, the reduction factors associated with stub-column tests, equal in both EU (Q_{EU}^N) and US (Q_{US}^N) codes, and with bending tests along the two principal axes required by the EU design codes (Q_{EU}^{My} and Q_{EU}^{Mz}) are also reported. It should be noted that this upright cross-section choice allows for a quite exhaustive overview of the cases most frequently encountered in routine rack design, with the ratio between the second moments of area ranging from 1.0 to 3.0 and the ratio associated with section moduli from 1.0 to 1.5, approximately. Pallet beams are comprised of rectangular hollow sections (100 × 50 × 3 mm RHS) and square hollow sections (35 × 35 × 2 mm SHS) are used for the lacings of the upright frames. All these structural components are in S355 steel grade [10], with a yielding strength of 355 Mpa.

For each of these uprights, four rack configurations have been defined, differing in the number of load levels (LL) and the inter-storey height (h_i): two (_2LL with $h_i=2500$ mm), three (_3LL with $h_i=2250$ mm), four (_4LL with $h_i=1800$ mm) and five (_5LL with



Fig. 3. Detail of the beam-to-column connection.

$h_i=1500$ mm) storeys (Fig. 1). Only the case of fully loaded racks has been considered with pallet units acting as uniform load on pallet beams. Furthermore, attention has been focussed on the following parameters:

- the degree of flexural stiffness associated with beam-to-column joints. In particular, several values of rotational stiffness ($S_{j,btc}$), of interests for practical applications have been selected and expressed as multiples (by means of term $\rho_{j,btc}$) of a reference stiffness value $S_{j,btc}^{EC3-LB}$ via the relation

$$S_{j,btc} = \rho_{j,btc} \cdot S_{j,btc}^{EC3-LB} \quad (1)$$

where $S_{j,btc}^{EC3-LB}$ is the stiffness associated with the lower bound of the semi-rigid domain, i.e. the value corresponding to the transition between the domains of flexible (pinned) and semi-rigid joints according to the classification criteria of part 1–8 of Eurocode 3 [11]. The parameter $\rho_{j,btc}$ has been assumed to range from 1 to 10, and in addition also the values of $\rho_{j,btc}$ equal to 2,4,5 and 8 have been considered.

- the degree of flexural stiffness associated with base-plate connections. As for beam-to-column joints, the values of the base rotational stiffness $S_{j,base}$ have been selected as multiples, by means of term $\rho_{j,base}$, of the upper transition stiffness ($S_{j,base}^{EC3-UB}$) between the region of semi-rigid and rigid joints, defined as

$$S_{j,base} = \rho_{j,base} \cdot S_{j,base}^{EC3-UB} \quad (2)$$

Three values have been considered ($\rho_{j,base}=0.15$, $\rho_{j,base}=0.30$ and $\rho_{j,base}=0.45$) to characterize the rotational behaviour of the base-plate connections. It worth mentioning that according to RMI specifications, no experimental tests for the determination of the base-plate flexural stiffness are required for rack manufacturers: an analytic expression is provided, which underestimates the real base-plate joint stiffness, as shown by Sarawit and Pekoz [4]. In the present parametric study, the authors used a unique value for the EU and US design procedures, mainly because when EU manufacturers have the results of the base-plate test, designers prefer to use them, which are more accurate than other more general theoretical approaches.

In order to propose design cases that are comparable to one another and research outcomes of interest for researchers and designers, preliminarily to the design phase, a buckling analysis has been carried out for each rack. In particular, the design pallet load $q = \alpha_s q_c$, is defined as 0.56 times the value of the elastic critical load multiplier ($\alpha_s = 0.56 \cdot \alpha_{cr}$), i.e. the ratio between the critical vertical load and the applied load on each rack is approximately 1.8. Buckling and second-order elastic analyses have been carried out by means of the commercial finite element analysis package ConSteel [12], characterized by a very refined beam formulation able to account for warping effects, outside the scope of the present research. Furthermore, it is worth mentioning that any other commercial programme should be able to reproduce efficiently the proposed research outcomes, owing to the quite limited degree of refinement required for the beam formulation for the modelling of bi-symmetric cross-section components.

The layout summarizing the key parameters considered in this study is represented in Fig. 4: in total 2160 design analyses have been carried out on 216 racks by applying four EU approaches, each of them appraising the effects of imperfections via both notional loads and imperfect rack models, and two US design procedures.

Owing to the large amount of data and the need to identify clearly the research outcomes, the numerical study has been carried out focusing attention on the uprights only, neglecting joint and pallet beam verifications. Furthermore, to allow for a direct comparison between the considered design approaches, for each rack, reference has been made to the maximum value of the upright safety index (SI^{j-k}) defined as

$$SI^{j-k} = \sum_{i=1}^N \frac{E_d^{j-k}}{R_d^{j-k}} \leq 1 \quad (3)$$

where E_d is the design value of the axial force or bending moment, R_d is the associated resistance, N is the number of terms to be used in

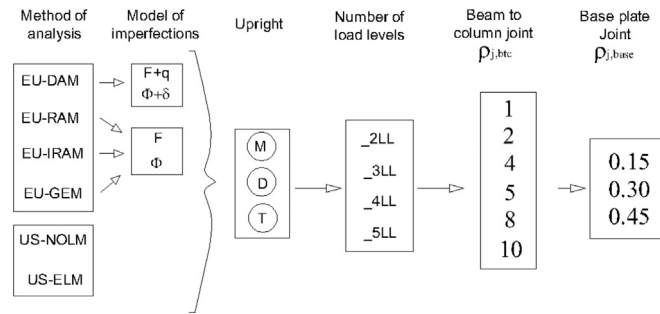


Fig. 4. Layout of the parametric study.

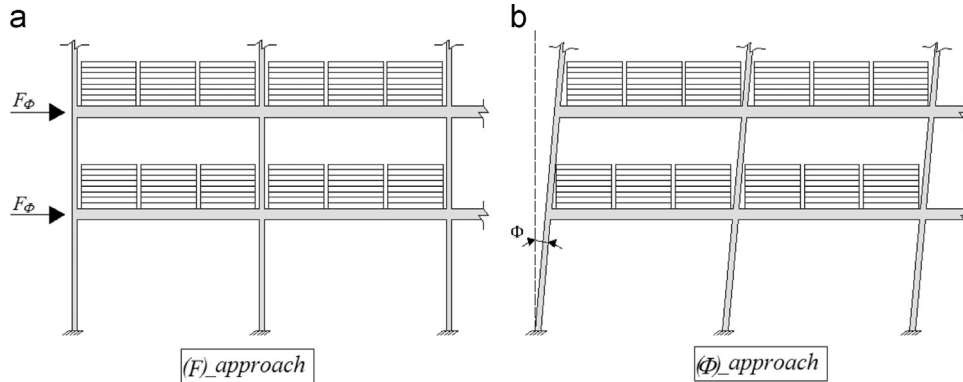


Fig. 5. Different method for modelling global sway imperfections (a) the notional load and (b) the deformed rack approach.

the verification equation and superscripts j and k identify the design code and the considered approach, respectively.

3. Application of the Eu alternatives

Two types of imperfections should be considered in pallet rack design as well as for more traditional steel framed buildings: overall frame (sway) imperfections and out-of-straightness (bow) member imperfections. The former are associated with an out-of-plumb angle of the uprights, Φ expressed in radians as

$$\Phi = \Phi_s + \Phi_l \geq 1/500 \quad (4)$$

where the angle Φ_s is the maximum specified out-of-plumb displacement divided by the height and the angle Φ_l accounts for the looseness of the beam-upright connector, determined according to the standard provisions.

The angle Φ_l depends on the details of the beam-to-column joints, which has to be evaluated on the basis of the initial slippage of the experimental moment-rotation joint curve. Owing to the need to limit the number of variables affecting research outcomes, this contribution has been neglected for all cases in the present study.

A constant angle, Φ_s , has been assumed for each rack equal to $\Phi_s = \frac{1}{357} = 0.0028$ rad.

As to the modelling techniques to be adopted in structural analysis, sway imperfections could be accounted for either via notional horizontal forces concentrated at the floor levels (Fig. 5a) or via inclined uprights (Fig. 5b),

With reference to the member imperfections, it is worth mentioning that the EU rack code does not give clear indications, remanding to the EN1993-1-1 [7], which allows these imperfections to be simulated via equivalent distributed loads or via the direct simulation of curved members. Defining e_0 the maximum out-of-straightness defect with respect to the ideal configuration (Fig. 6), it is possible to make reference to an equivalent uniformly distributed load of magnitude q_δ defined as:

$$q_\delta = \frac{8e_0N_{Ed}}{L^2} \quad (5)$$

where L is the length of the member subjected to the design axial forces N_{Ed} .

This approach is proposed with reference to an isolated member and hence its direct extension to the spatial and regular framed systems, such as storage pallet racks, does not seem to have a unique interpretation. Otherwise, it is worth mentioning that the Australian rack code [13] is more exhaustive on this topic and allows designers to limit the member imperfection effects to the first two floors of the rack, as underlined also by Rasmussen and Benoit [14]. The effects of bow imperfections on rack frames should hence be taken into account via the scheme presented in Fig. 7.

At first, attention has been focused on the influence that the imperfection modelling technique has on the value of internal forces and bending moments. For the sake of simplicity, as shown in Fig. 8, the $(F+q)$ and $(\Phi+\delta)$ tags identify the equivalent load and the deformed rack approaches, respectively, when both sway and member imperfections have to be accounted for. Similarly, when the only sway

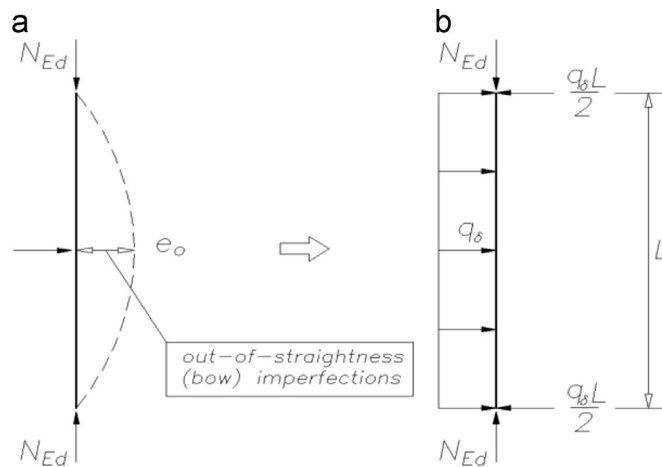


Fig. 6. Horizontal forces equivalent to the out-of-straightness imperfection (bow) according to EC3 [7].

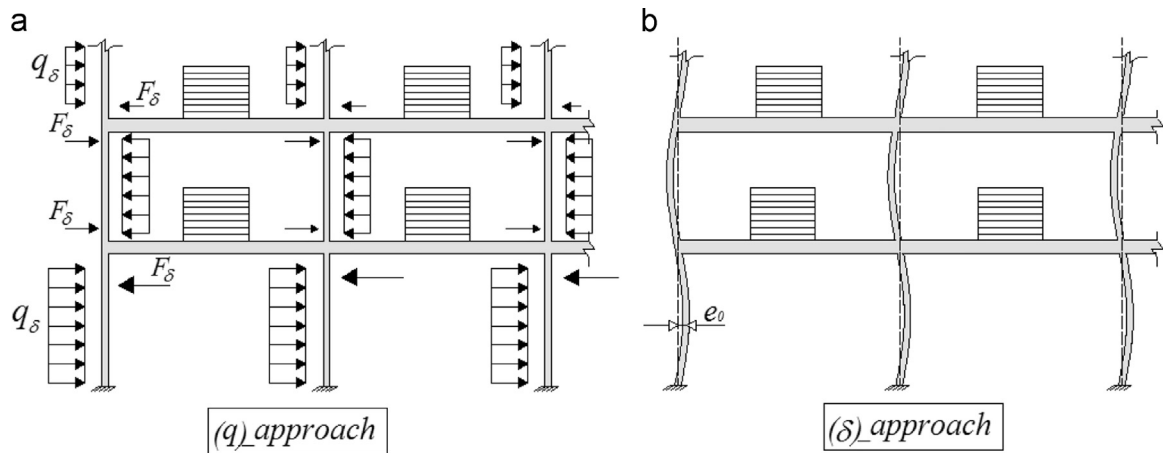


Fig. 7. Different method for modelling local (member) imperfections (a) the notional load and (b) the deformed member approach.

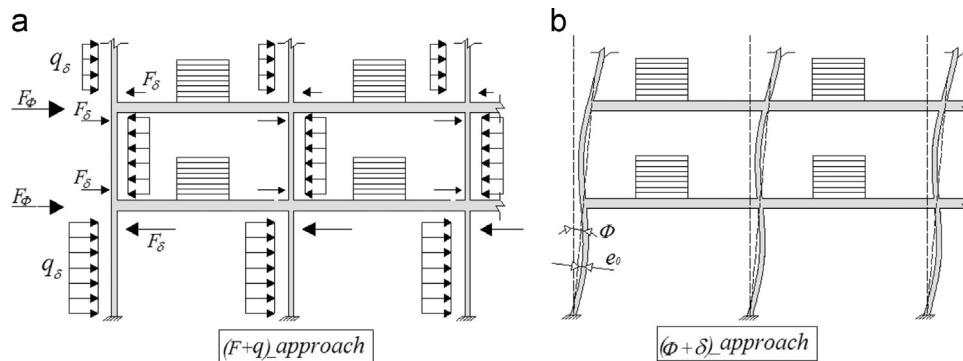


Fig. 8. Different method for modelling local (member) imperfections (a) the notional load and (b) the deformed member approach.

imperfections should be considered, (F) and (Φ) terms are used to identify the corresponding approaches, already shown in parts a) and b), respectively, of Fig. 5. On the basis of the numerical results associated with the considered design paths, it has been noted that the value of the axial load on the most stressed upright is independent of the approach used to account for imperfections. In particular, in the case of imperfect racks, $(\Phi+\delta)_\text{approach}$ and $(\Phi)_\text{approach}$, the values of the axial load are very slightly lower than the ones obtained via notional loads $(F+q)_\text{approach}$ and $(F)_\text{approach}$ and differences are never greater than 0.1%, i.e. they are negligible from an engineering point of view. Much more significant differences are related to the bending moments in the down-aisle direction, as it appears from the values of the $\frac{M_{y,F+q}}{M_{y,\Phi+\delta}}$ and the $\frac{M_{y,F}}{M_{y,\Phi}}$ ratios, plotted in Figs. 9 and 10, respectively versus $\rho_{j,btc}$; reference has been made to the most stressed internal upright for each rack, considering the maximum bending moment that is always at the base joint location. As to the data associated with the modelling of both types of imperfections, it can be noted that the ratio $\frac{M_{y,F+q}}{M_{y,\Phi+\delta}}$ is always lower than unity for two and three load level racks; otherwise,

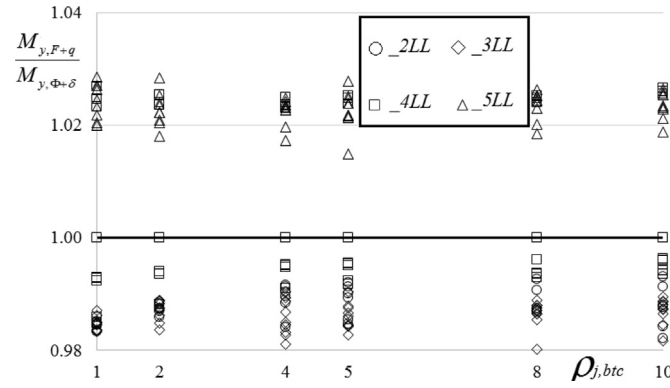


Fig. 9. Influence of the sway and member imperfection modelling ($\frac{M_{y,F+q}}{M_{y,\Phi+\delta}}$).

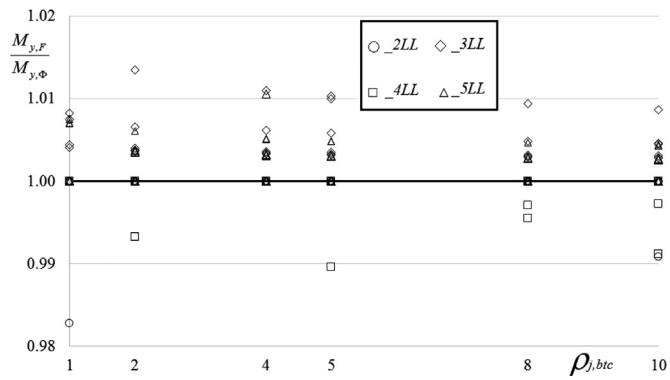


Fig. 10. Influence of the sway frame imperfection modelling ($\frac{M_{y,F}}{M_{y,\Phi}}$).

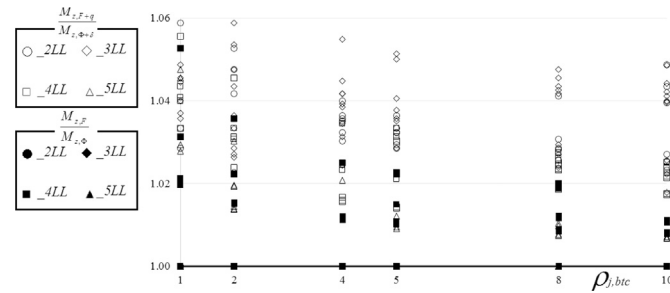


Fig. 11. Influence of the imperfection modelling approaches in term of $\frac{M_{z,F}}{M_{z,\Phi}}$ (filled point) and $\frac{M_{z,F+q}}{M_{z,\Phi+\delta}}$ (empty point).

this ratio is greater than unity for the lowest value of the inter-storey height (5LL). However, it should be remarked that, independent of the joint stiffness, the data have a very moderate dispersion, falling in the range of $0.98 \div 1.03$ with a mean value of 0.998 and a standard deviation of 0.015. In the case of sway imperfections only, the representative points of the $\frac{M_{y,F}}{M_{y,\Phi}}$ ratio plotted in Fig. 10 showed that in general the use of the notional loads leads to slightly greater values of bending moment with the exception of a very limited number of cases associated with the analysis of the two and four load level racks: also in this case the dispersion of $\frac{M_{y,F}}{M_{y,\Phi}}$ is very moderate, with a mean value of 1.001 and a standard deviation of 0.003.

The influence of the approach to model imperfections on the bending moments along the cross-aisle direction (M_z) is quite greater than that observed for moments along the down-aisle direction (M_y). In Fig. 11 the $\frac{M_{z,F+q}}{M_{z,\Phi+\delta}}$ and $\frac{M_{z,F}}{M_{z,\Phi}}$ ratios are plotted versus the beam-to-column non-dimensional stiffness. In these cases, all the data are never lower than unity. Considering both imperfection types, the dispersion is much greater than when only bow imperfections are considered. The mean value of the $\frac{M_{z,F+q}}{M_{z,\Phi+\delta}}$ ratio is 1.026, with a maximum of 1.06 and a standard deviation of 0.018; with reference to the $\frac{M_{z,F}}{M_{z,\Phi}}$ ratio the maximum and mean values are 1.053 and 1.002, respectively, with a standard deviation of 0.006.

It should be noted that different values in the bending moments associated with the use of notional loads and imperfect racks have been found only in a very limited number of cases, not relevant from the design point of view, as it appears from the values of the safety index (SI) associated with the different approaches to model imperfections. Table 2 summarises the $\frac{SI_{F+q}^{EU-DAM}}{SI_{\Phi+\delta}^{EU-DAM}}$, $\frac{SI_F^{EU-RAM}}{SI_{\Phi}^{EU-RAM}}$, $\frac{SI_{F+q}^{EU-IRAM}}{SI_{\Phi}^{EU-IRAM}}$ and $\frac{SI_F^{EU-GEM}}{SI_{\Phi}^{EU-GEM}}$

Table 1

Key features of the considered cross-section for the uprights (US term in bracket).

	M _–	D _–	T _–
A [mm ²]	780	1000	1220
(S _x) W _y [mm ³]	24.9 · 10 ³	35.5 · 10 ³	46.1 · 10 ³
(S _y) W _z [mm ³]	24.9 · 10 ³	44.7 · 10 ³	68.6 · 10 ³
(I _x) I _y [mm ⁴]	124.6 · 10 ⁴	177.5 · 10 ⁴	230.3 · 10 ⁴
(I _y) I _z [mm ⁴]	124.6 · 10 ⁴	346.6 · 10 ⁴	719.9 · 10 ⁴
(J) I _t [mm ⁴]	191.3 · 10 ⁴	362.8 · 10 ⁴	548.8 · 10 ⁴
(C _w) I _w [mm ⁶]	55.7 · 10 ⁴	2.36 · 10 ⁸	14.2 · 10 ⁸
$Q^N = Q_{EU}^N = Q_{US}^N$		0.850	
Q_{EU}^{My}		0.925	
Q_{EU}^{Mz}		0.925	

Table 2

Influence of the imperfection modelling according to the EU approaches.

		$\frac{SI_{F+Q}^{EU-DAM}}{SI_{\phi+\delta}^{EU-DAM}}$	$\frac{SI_F^{EU-RAM}}{SI_{\phi}^{EU-RAM}}$	$\frac{SI_F^{EU-IRAM}}{SI_{\phi}^{EU-IRAM}}$	$\frac{SI_F^{EU-GEM}}{SI_{\phi}^{EU-GEM}}$
M_2LL	Mean	1.000	1.000	1.000	1.000
D_2LL	St. dev	0.0012	0.0005	0.0004	0.0002
T_2LL	Min	0.997	0.997	0.998	0.999
	Max	1.002	1.001	1.002	1.000
M_3LL	Mean	1.000	1.003	1.000	1.000
D_3LL	St. dev	0.0013	0.0034	0.0004	0.0004
T_3LL	Min	0.997	1.000	0.999	1.000
	Max	1.002	1.008	1.002	1.002
M_4LL	Mean	1.000	1.000	1.000	1.000
D_4LL	St. dev	0.0008	0.0010	0.0008	0.0006
T_4LL	Min	0.997	0.999	0.998	0.998
	Max	1.002	1.003	1.002	1.002
M_5LL	Mean	1.004	1.000	1.000	1.000
D_5LL	St. dev	0.0011	0.0004	0.0002	0.0002
T_5LL	Min	1.002	1.000	1.000	1.000
	Max	1.008	1.001	1.001	1.001
All	Mean	1.001	1.001	1.000	1.000
	St. dev	0.0021	0.0022	0.0005	0.0004
	Min	0.997	0.997	0.998	0.998
	Max	1.008	1.008	1.002	1.002

ratios, presenting the mean (*mean*), standard deviation (*st dev*), minimum (*min*) and maximum (*Max*) values for each set of frames with the same number of load levels. The influence of the approaches to model bow and member imperfection types is extremely limited, differences in the SI values are always lower than 1%. Also in the cases of the sole sway frame imperfections (EU-RAM, EU-IRAM and EU-GEM approaches), the use of the notional loads or curved members leads to very moderate differences, never greater than 1%. As a consequence, it can be concluded that the technique to model the imperfection effects has a very limited influence on the design, also owing to the fact that the axial load always plays a dominant role with respect to the ones associated with bending moments. Consequently, considering that the notional load approaches are the ones preferred by designers and lead in general to more conservative results in terms of load carrying capacity, in the following the data related to geometrically imperfect racks have not been considered for the proposed outlines related to the methods of analysis.

Key data associated with the EU notional load approaches, have been at first treated separately on the basis of the upright cross-section type and presented in Table 3 (M_racks), 4 (D_racks) and 5 (T_racks) together with the ratio between the maximum and the minimum value of the SI (*Max/min*). Furthermore, by identifying with SI^{EU-Max} the maximum SI value associated with each rack and with SI^{EU-k} the one corresponding to the k^{th} approach, the $\frac{SI^{EU-Max}}{SI^{EU-k}}$ ratio has been plotted, for all the considered racks, in Fig. 12. From these data it can be noted that:

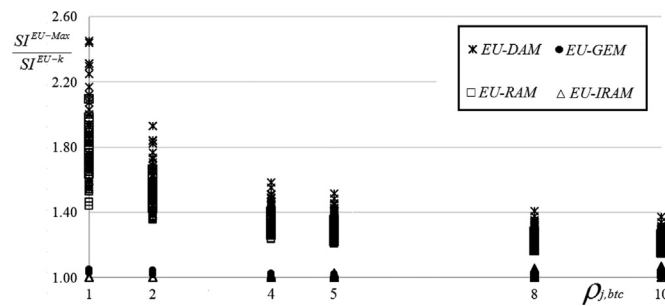
- Independent of the frame geometry and on the degree of stiffness of beam-to-column joints and base-plate connections, the minimum value of the safety index is always associated with the EU-DAM;
- the maximum SI values are associated with the EU-IRAM or the EU-GEM approaches, and their differences are however limited, lower than 7% for the M_racks and 5% for the D_– and T_–racks, as it appears from Fig. 13 which shows the same data of Fig. 12 but excluding the EU-DAM and EU-RAM approaches;

Table 3

Values of the safety index (SI) according to the European approaches (imperfection modelled via notional load), for the M_racks.

$\rho_{j,base}$	$\rho_{j,btc}$	M_{2LL}					$\rho_{j,base}$	$\rho_{j,btc}$	M_{3LL}				
		EU-DAM	EU-RAM	EU-IRAM	EU-GEM	Max/min			EU-DAM	EU-RAM	EU-IRAM	EU-GEM	Max/min
0.15	1	0.33	0.41	0.81	0.78	2.44	0.15	1	0.33	0.39	0.81	0.77	2.45
	2	0.45	0.54	0.87	0.85	1.93		2	0.46	0.54	0.89	0.86	1.93
	4	0.60	0.69	0.93	0.94	1.58		4	0.63	0.70	0.97	0.97	1.55
	5	0.65	0.73	0.95	0.98	1.51		5	0.68	0.76	0.99	1.01	1.48
	8	0.74	0.82	0.99	1.04	1.41		8	0.80	0.86	1.04	1.10	1.38
0.30	10	0.78	0.85	1.01	1.07	1.37	0.30	10	0.85	0.90	1.07	1.14	1.34
	1	0.37	0.45	0.84	0.80	2.30		1	0.36	0.42	0.83	0.79	2.31
	2	0.49	0.60	0.91	0.88	1.84		2	0.50	0.59	0.93	0.89	1.84
	4	0.66	0.77	0.99	1.00	1.51		4	0.69	0.78	1.03	1.02	1.49
	5	0.72	0.82	1.02	1.04	1.45		5	0.75	0.84	1.06	1.07	1.42
0.45	8	0.83	0.92	1.07	1.12	1.35	0.45	8	0.89	0.96	1.13	1.18	1.33
	10	0.88	0.96	1.09	1.15	1.32		10	0.95	1.02	1.16	1.23	1.29
	1	0.38	0.47	0.85	0.81	2.25		1	0.36	0.42	0.83	0.79	2.31
	2	0.51	0.62	0.93	0.89	1.82		2	0.51	0.59	0.93	0.89	1.82
	4	0.68	0.80	1.01	1.02	1.48		4	0.69	0.79	1.03	1.03	1.49
	5	0.74	0.86	1.04	1.06	1.43		5	0.76	0.85	1.07	1.08	1.42
	8	0.86	0.96	1.10	1.15	1.33		8	0.90	0.98	1.14	1.19	1.32
	10	0.91	1.01	1.12	1.19	1.30		10	0.96	1.04	1.18	1.24	1.29

$\rho_{j,base}$	$\rho_{j,btc}$	M_{4LL}					$\rho_{j,base}$	$\rho_{j,btc}$	M_{5LL}				
		EU-DAM	EU-RAM	EU-IRAM	EU-GEM	Max/min			EU-DAM	EU-RAM	EU-IRAM	EU-GEM	Max/min
0.15	1	0.39	0.43	0.85	0.81	2.17	0.15	1	0.45	0.48	0.88	0.85	1.97
	2	0.55	0.61	0.96	0.93	1.73		2	0.65	0.68	1.02	1.00	1.58
	4	0.77	0.83	1.10	1.09	1.44		4	0.90	0.95	1.22	1.21	1.35
	5	0.84	0.90	1.15	1.15	1.37		5	1.00	1.04	1.29	1.29	1.30
	8	1.00	1.05	1.25	1.29	1.28		8	1.20	1.23	1.43	1.47	1.23
0.30	10	1.08	1.11	1.29	1.35	1.25	0.30	10	1.29	1.31	1.49	1.55	1.20
	1	0.41	0.45	0.86	0.83	2.10		1	0.47	0.49	0.89	0.86	1.92
	2	0.59	0.64	0.98	0.95	1.67		2	0.67	0.71	1.04	1.02	1.54
	4	0.82	0.89	1.16	1.13	1.42		4	0.95	0.99	1.26	1.25	1.33
	5	0.90	0.96	1.21	1.20	1.35		5	1.04	1.09	1.34	1.33	1.29
0.45	8	1.07	1.13	1.32	1.35	1.26	0.45	8	1.26	1.30	1.50	1.53	1.21
	10	1.15	1.20	1.37	1.42	1.23		10	1.36	1.39	1.57	1.61	1.19
	1	0.42	0.46	0.87	0.83	2.07		1	0.47	0.50	0.90	0.87	1.89
	2	0.60	0.65	0.99	0.96	1.66		2	0.68	0.72	1.05	1.02	1.54
	4	0.83	0.90	1.17	1.14	1.41		4	0.96	1.00	1.27	1.26	1.32
	5	0.91	0.99	1.23	1.21	1.34		5	1.06	1.11	1.35	1.35	1.28
	8	1.09	1.16	1.35	1.37	1.25		8	1.28	1.32	1.53	1.54	1.20
	10	1.18	1.23	1.40	1.44	1.22		10	1.39	1.42	1.60	1.64	1.18

**Fig. 12.** Comparison between the EU design approaches in terms of $\frac{SI^{EU-Max}}{SI^{EU-k}}$ ratio.

- the SI^{EU-RAM} is slightly greater than SI^{EU-DAM} and always lower than the safety index associated with the other approaches. The differences between the EU-RAM and EU-DAM approaches are non-negligible only in a very limited number of cases, up to 1.24 for M_, 1.13 for D_ and 1.09 for T_ racks in cases of two load levels and never lower than 1.01;
- remarkable differences can be observed by comparing all together the SI values. The ratio $\frac{SI^{EU-Max}}{SI^{EU-min}}$ (with $SI^{EU-min} = SI^{EU-DAM}$) decreases with the increase of the degree of beam-to-column joint stiffness and it is very moderately influenced by the flexural stiffness of the base-plate connections: increasing $\rho_{j,base}$, the $\frac{SI^{EU-Max}}{SI^{EU-min}}$ ratio decreases slightly;
- the values of the $\frac{SI^{EU-Max}}{SI^{EU-min}}$ ratio is significantly influenced by the cross-section geometry, ranging from 2.45 (M_3LL racks with a $\rho_{j,btc} = 1.0$ and $\rho_{j,base} = 0.15$) to 1.18 (T_5LL racks with a $\rho_{j,btc} = 10.0$ and $\rho_{j,base} = 0.45$) with mean value of 1.55 (M_racks), 1.47 (D_racks) and 1.39 (T_racks).



Fig. 13. Comparison between the EU approaches in terms of $\frac{SI^{EU-Max}}{SI^{EU-IRAM}}$ and $\frac{SI^{EU-Max}}{SI^{EU-GEM}}$ ratios.

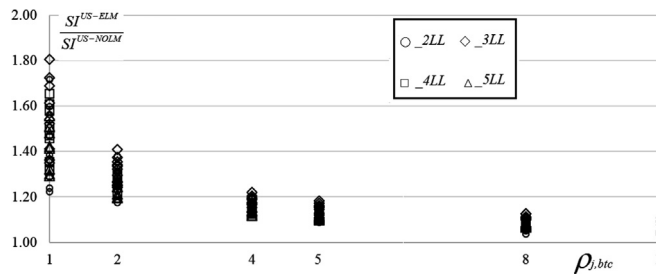


Fig. 14. Comparison between the US approaches in terms of $\frac{SI^{US-ELM}}{SI^{US-NOLM}}$ ratio.

As a preliminary conclusion, it can be stated that a very important open question is if the EU-DAM and EU-RAM approaches are adequate or not for rack design. In the first case, the EU-IRAM and the EU-GEM alternatives appear excessively conservative, leading to very heavy racks. Otherwise, the degree of reliability of the racks designed according to EU-DAM and EU-RAM approaches are significantly under-estimated, leading unsafe racks being introduced into the market. At this stage, no answer seems possible based on the present data but it should be underlined that, in general, the proposal of design alternatives leading to such different SI values appears unacceptable and misleading from the design point of view.

4. Application of the Us alternatives

The US sway imperfections have always been modelled in the present study, as notional loads evaluated on the basis of a story out-of-plumbness of 0.5 in. (12.7 mm) in 10 feet (3.05 m), which represents an out-of-plumb angle of approximately 0.0042 rad ($\Phi = \frac{1}{240}$ rad) according to the maximum fabrication and erection tolerances admitted by the RMI specifications [2]. As reported by Sarawit and Pekoz [4], two US design alternatives are currently offered: the US-ELM and the US-NOLM approaches. Both have been applied to the considered racks: from the numerical results it can be noted that the US-ELM approach is always the more conservative one, independent of the key parameters considered in the analysis: for this reason Table 6 reports the $\frac{SI^{US-ELM}}{SI^{US-NOLM}}$ and the $\frac{SI^{US-ELM}}{SI^{US-NOLM}}$ values. It is worth mentioning that EU and US code adopt different symbols, therefore to better identify the parameters governing design, reference can be made to Table B1. Furthermore, Fig. 14 presents the values of the $\frac{SI^{US-ELM}}{SI^{US-NOLM}}$ ratio plotted versus $\rho_{j,btc}$ grouped in four sets of data, each of them associated with one of the considered longitudinal configurations (Fig. 1). In particular, it can be noted that the two US approaches lead to quite different values of the safety index: $\frac{SI^{US-ELM}}{SI^{US-NOLM}}$ ratios decrease with the increases of the $\rho_{j,btc}$ and are approximately independent of the value of the degree of stiffness of base-plate connections. The values of ratio $\frac{SI^{US-ELM}}{SI^{US-NOLM}}$ are very high, especially when $\rho_{j,btc} = 1.0$ and $\rho_{j,base} = 0.15$, ranging from 1.32 (T_5LL) up to 1.81 (M_3LL). Increasing $\rho_{j,btc}$ the differences decrease and for $\rho_{j,btc} \geq 4.0$ the ratio is never greater than 1.2. It should be noted that these differences between the US methods are comparable with the ones found in a previous numerical study [15] on the design approaches admitted by the RMI specifications.

5. Comparative analysis of the design results

A direct comparison between the SI values associated with both EU (Tables 3–5) and US (Table 6) approaches clearly identifies that two different sets of SI values can be appraised, which define two domains of data that are remarkably different and never overlapped, as shown in Figs. 15 (M_racks), 16 (D_racks) and 17 (T_racks). The upper region (A) is associated with the EU-DAM and EU-RAM approaches and the lower (B) defines the domain where the two US and the other two EU approaches representative points are located. It clearly appears that the EU-DAM and EU-RAM methods always lead to a significant over-estimation of the rack performance, differing greatly from the US results. Furthermore, owing to the fact that the EU-IRAM and EU-GEM approaches lead to rack performances not so different from the US ones, it is opinion of the authors that both EU-DAM and EU-RAM approaches need to be urgently calibrated or removed from the EU design options. Neither of these methods can not be used in the present form, as confirmed also by the relative frequency of the

Table 4

Values of the safety index (SI) in according to the European approaches (imperfection modelled via notional load), for the D_racks.

$\rho_{j,base}$	$\rho_{j,btc}$	D_{2LL}					$\rho_{j,base}$	$\rho_{j,btc}$	D_{3LL}				
		EU-DAM	EU-RAM	EU-IRAM	EU-GEM	Max/min			EU-DAM	EU-RAM	EU-IRAM	EU-GEM	Max/min
0.15	1	0.45	0.51	0.90	0.86	2.00	0.15	1	0.41	0.45	0.87	0.83	2.12
	2	0.56	0.63	0.98	0.95	1.74		2	0.54	0.59	0.96	0.92	1.77
	4	0.73	0.80	1.10	1.07	1.51		4	0.73	0.78	1.10	1.07	1.51
	5	0.79	0.86	1.15	1.12	1.45		5	0.80	0.85	1.15	1.13	1.45
	8	0.92	0.99	1.24	1.23	1.34		8	0.94	1.00	1.27	1.25	1.34
0.30	10	0.98	1.05	1.27	1.28	1.31	0.30	10	1.01	1.07	1.31	1.32	1.30
	1	0.50	0.56	0.94	0.90	1.89		1	0.44	0.48	0.89	0.85	2.03
	2	0.62	0.69	1.02	0.99	1.66		2	0.58	0.62	0.98	0.95	1.71
	4	0.79	0.88	1.16	1.13	1.46		4	0.77	0.83	1.14	1.11	1.47
	5	0.86	0.94	1.21	1.19	1.41		5	0.85	0.91	1.20	1.17	1.41
0.45	8	1.00	1.09	1.33	1.31	1.32	0.45	8	1.01	1.08	1.33	1.32	1.32
	10	1.07	1.16	1.38	1.37	1.28		10	1.09	1.15	1.40	1.38	1.29
	1	0.51	0.58	0.95	0.91	1.85		1	0.45	0.49	0.90	0.86	2.00
	2	0.64	0.71	1.04	1.01	1.63		2	0.59	0.64	0.99	0.96	1.69
	4	0.82	0.90	1.18	1.15	1.44		4	0.79	0.85	1.15	1.13	1.45
	5	0.89	0.97	1.23	1.21	1.39		5	0.87	0.93	1.21	1.19	1.40
	8	1.04	1.13	1.36	1.34	1.31		8	1.03	1.10	1.35	1.34	1.31
	10	1.11	1.20	1.41	1.40	1.28		10	1.11	1.18	1.42	1.41	1.28

$\rho_{j,base}$	$\rho_{j,btc}$	D_{4LL}					$\rho_{j,base}$	$\rho_{j,btc}$	D_{5LL}				
		EU-DAM	EU-RAM	EU-IRAM	EU-GEM	Max/min			EU-DAM	EU-RAM	EU-IRAM	EU-GEM	Max/min
0.15	1	0.47	0.49	0.90	0.87	1.94	0.15	1	0.52	0.54	0.94	0.91	1.80
	2	0.63	0.66	1.02	0.99	1.62		2	0.72	0.75	1.09	1.07	1.51
	4	0.87	0.91	1.21	1.19	1.40		4	1.00	1.03	1.32	1.31	1.32
	5	0.95	0.99	1.29	1.27	1.35		5	1.10	1.13	1.41	1.40	1.28
	8	1.14	1.19	1.45	1.44	1.27		8	1.33	1.36	1.61	1.61	1.22
0.30	10	1.23	1.28	1.53	1.52	1.25	0.30	10	1.43	1.47	1.71	1.71	1.20
	1	0.49	0.52	0.92	0.89	1.88		1	0.55	0.56	0.96	0.92	1.75
	2	0.67	0.70	1.05	1.02	1.58		2	0.75	0.78	1.11	1.09	1.48
	4	0.91	0.95	1.25	1.23	1.37		4	1.04	1.07	1.35	1.34	1.30
	5	1.00	1.04	1.33	1.31	1.33		5	1.15	1.18	1.45	1.44	1.26
0.45	8	1.20	1.25	1.50	1.49	1.25	0.45	8	1.38	1.41	1.66	1.66	1.20
	10	1.29	1.34	1.59	1.58	1.23		10	1.49	1.53	1.76	1.77	1.18
	1	0.50	0.53	0.93	0.89	1.85		1	0.55	0.57	0.96	0.93	1.74
	2	0.68	0.71	1.06	1.03	1.56		2	0.76	0.79	1.12	1.10	1.47
	4	0.92	0.96	1.26	1.24	1.36		4	1.05	1.08	1.37	1.36	1.30
	5	1.02	1.06	1.34	1.33	1.32		5	1.16	1.19	1.46	1.45	1.26
	8	1.22	1.27	1.52	1.51	1.25		8	1.40	1.43	1.68	1.68	1.20
	10	1.32	1.37	1.61	1.60	1.22		10	1.51	1.55	1.78	1.78	1.18

$\frac{SI^{Max}}{SI^{EU-RAM}}$ and $\frac{SI^{Max}}{SI^{EU-DAM}}$ ratios presented in Fig. 18. Neglecting in fact the right queue with several values greater than two, a large amount of data ranges from 1.3 to 1.7. For this reason, these approaches should be urgently excluded from design options and Table 7 presents the ratio between the maximum safety index, which is always associated with the US-ELM approach and the ones according to EU-IRAM, EU-GEM, US-NOLM approaches. It is worth mentioning that there is no experimental data currently available to select the most convenient design approaches. In refs. [4,15] it has been concluded that the US-ELM best fits the results of very refined non-linear numerical analyses and hence it seems reasonable to state that the best prediction of the frame performance can be associated with EU-IRAM, EU-GEM and US-ELM. In Fig. 19, the relative frequency of the $\frac{SI^{US-ELM}}{SI^{EU-IRAM}}$ and $\frac{SI^{US-ELM}}{SI^{EU-GEM}}$ is represented; it can be noted that the values of the SI index associated with these methods are very close, confirming their adequacy for rack design. Only in a very limited number of cases (less than 10%) differences are greater than 10% but lower than 16%, confirming the equivalence, from a design point of view of EU-IRAM and EU-GEM with the US-ELM.

6. Conclusions

The methods of structural analysis and design permitted by the European and United States provisions for medium-rise pallet racks have been applied in this paper, which summarises a more general study discussed in the two-part paper. In particular, four EU and two US approaches have been considered and applied. A parametric study has been based on 2160 design cases on racks differing in geometry, model imperfection technique, load conditions and degree of rotational stiffness of both beam-to-column joints and base-plate connections, allowing for a direct comparison in terms of load-carrying capacity. Research outcomes are gained for bi-symmetric cross-section upright but maintain their validity also in the case of mono- or non-symmetric cross-sections. As discussed in the paper, attention has in

Table 5

Values of the safety index (SI) according to the European approaches (imperfection modelled via notional load), for the T_racks.

$\rho_{j,base}$	$\rho_{j,btc}$	T_2LL					$\rho_{j,base}$	$\rho_{j,btc}$	T_3LL				
		EU-DAM	EU-RAM	EU-IRAM	EU-GEM	Max/min			EU-DAM	EU-RAM	EU-IRAM	EU-GEM	Max/min
0.15	1	0.61	0.67	1.03	1.00	1.68	0.15	1	0.52	0.55	0.95	0.91	1.83
	2	0.71	0.77	1.10	1.08	1.55		2	0.64	0.67	1.04	1.00	1.63
	4	0.86	0.92	1.22	1.20	1.42		4	0.81	0.86	1.18	1.15	1.45
	5	0.92	0.98	1.27	1.25	1.39		5	0.88	0.93	1.23	1.21	1.40
	8	1.05	1.11	1.38	1.37	1.32		8	1.03	1.08	1.36	1.35	1.32
	10	1.11	1.18	1.44	1.43	1.29		10	1.11	1.16	1.43	1.42	1.29
0.30	1	0.68	0.74	1.09	1.06	1.60	0.30	1	0.56	0.59	0.98	0.94	1.75
	2	0.78	0.85	1.17	1.14	1.49		2	0.68	0.72	1.07	1.04	1.57
	4	0.94	1.01	1.30	1.28	1.38		4	0.87	0.91	1.22	1.20	1.41
	5	1.00	1.07	1.35	1.33	1.35		5	0.94	0.98	1.28	1.26	1.37
	8	1.14	1.22	1.47	1.46	1.29		8	1.10	1.15	1.43	1.41	1.30
	10	1.21	1.29	1.54	1.52	1.27		10	1.18	1.23	1.50	1.48	1.27
0.45	1	0.71	0.77	1.11	1.08	1.57	0.45	1	0.57	0.61	0.99	0.95	1.73
	2	0.81	0.88	1.19	1.17	1.47		2	0.70	0.74	1.08	1.06	1.55
	4	0.97	1.04	1.33	1.31	1.37		4	0.89	0.93	1.24	1.22	1.40
	5	1.03	1.11	1.38	1.36	1.33		5	0.96	1.01	1.30	1.28	1.36
	8	1.18	1.25	1.51	1.50	1.28		8	1.12	1.18	1.45	1.43	1.29
	10	1.25	1.33	1.57	1.56	1.26		10	1.20	1.26	1.52	1.51	1.26
$\rho_{j,base}$	$\rho_{j,btc}$	T_4LL					$\rho_{j,base}$	$\rho_{j,btc}$	T_5LL				
		EU-DAM	EU-RAM	EU-IRAM	EU-GEM	Max/min			EU-DAM	EU-RAM	EU-IRAM	EU-GEM	Max/min
0.15	1	0.57	0.60	0.99	0.95	1.72	0.15	1	0.62	0.65	1.02	0.99	1.63
	2	0.72	0.76	1.10	1.07	1.52		2	0.81	0.84	1.16	1.14	1.44
	4	0.95	0.99	1.29	1.27	1.36		4	1.07	1.11	1.39	1.38	1.30
	5	1.03	1.08	1.36	1.35	1.32		5	1.17	1.22	1.49	1.48	1.27
	8	1.22	1.28	1.54	1.53	1.26		8	1.40	1.45	1.70	1.70	1.21
	10	1.31	1.38	1.62	1.61	1.23		10	1.51	1.56	1.80	1.80	1.19
0.30	1	0.61	0.64	1.01	0.98	1.67	0.30	1	0.65	0.68	1.04	1.01	1.59
	2	0.76	0.80	1.13	1.11	1.49		2	0.84	0.87	1.19	1.17	1.42
	4	0.99	1.04	1.33	1.31	1.34		4	1.11	1.15	1.43	1.42	1.28
	5	1.08	1.13	1.41	1.39	1.30		5	1.22	1.26	1.53	1.52	1.25
	8	1.28	1.34	1.59	1.58	1.24		8	1.46	1.50	1.75	1.75	1.20
	10	1.38	1.44	1.68	1.67	1.22		10	1.57	1.62	1.85	1.85	1.18
0.45	1	0.62	0.65	1.02	0.99	1.65	0.45	1	0.66	0.68	1.04	1.02	1.58
	2	0.78	0.81	1.14	1.12	1.47		2	0.85	0.88	1.20	1.18	1.41
	4	1.01	1.06	1.34	1.33	1.33		4	1.13	1.17	1.44	1.43	1.28
	5	1.10	1.15	1.42	1.41	1.30		5	1.24	1.27	1.54	1.53	1.25
	8	1.30	1.36	1.61	1.60	1.24		8	1.48	1.52	1.76	1.76	1.19
	10	1.40	1.46	1.70	1.69	1.22		10	1.59	1.64	1.87	1.87	1.18

fact been focussed mainly on the influence of the method of analysis, independent of the local cross-section behaviour. Furthermore, it should be noted that, in the case of non-bi-symmetric cross-section member, the discussed differences between the proposed results are expected to be extolled by the influence of warping effects, as confirmed by the preliminary results of research currently in progress [16].

From the discussed numerical results, it can be concluded that:

- the imperfection modelling technique has a negligible influence on the rack performance: very limited differences can be noted between the SI values associated with the use of notional loads and the modelling of non-perfect frames (inclined and curved columns). It appears that the first approach, that is the most commonly used by designers, results in slightly more conservative values and it is hence on the safe side;
- the EU-DAM and EU-RAM approaches lead to values of the SI significantly lower than the ones associated with the other approaches. As already mentioned, it is authors' opinion that, with both methods being derived from the design approaches for traditional steel framed structures, an accurate re-calibration seems necessary for their application to rack design. Consequently, the rack performance predicted via DAM and RAM approaches appears out of interest and validity for rack design, as confirmed by the proposed design;
- a quite accurate prediction of the rack performance seems possible via the EU-IRAM, EU-GEM and US-ELM approaches which are substantially equivalent to one another, leading to values of the load carrying capacity that are quite similar.

Furthermore, it is worth mentioning that the EU-GEM approach appears very promising for rack design not only because of its simplicity (no complex interaction factors nor design equations have to be used) but also for the possibility of including directly the contribution due to the bi-moment, that is of fundamental importance in the case of mono-symmetric cross-sections [6]. Furthermore, this approach appears to be an efficient alternative to the design assisted by testing when perforated members are used [17], owing to the possibility to evaluate resistance and stability performances considering the presence of regular perforations along the uprights via models processed by commercial finite element analysis packages.

Table 6

Values of the safety index (SI) in according to the United States approaches, for all the considered racks.

$\rho_{j,base}$	$\rho_{j,btc}$	M_{2LL}		D_{2LL}		T_{2LL}		M_{3LL}		D_{3LL}		T_{3LL}	
		S_I^{US-ELM}	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	S_I^{US-ELM}	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	S_I^{US-ELM}	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	S_I^{US-ELM}	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	S_I^{US-ELM}	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	S_I^{US-ELM}	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$
0.15	1	0.88	1.72	0.94	1.47	1.08	1.29	0.87	1.81	0.91	1.61	0.99	1.41
	2	0.92	1.38	1.02	1.31	1.17	1.23	0.93	1.41	0.99	1.37	1.09	1.30
	4	1.02	1.19	1.16	1.19	1.31	1.16	1.05	1.20	1.15	1.22	1.25	1.20
	5	1.06	1.15	1.21	1.16	1.36	1.14	1.09	1.16	1.22	1.18	1.31	1.17
	8	1.13	1.10	1.34	1.12	1.49	1.11	1.19	1.10	1.36	1.13	1.47	1.13
	10	1.16	1.08	1.40	1.10	1.55	1.10	1.24	1.08	1.43	1.11	1.54	1.11
	1	0.90	1.60	0.98	1.39	1.15	1.24	0.88	1.72	0.93	1.54	1.02	1.36
	2	0.96	1.31	1.07	1.26	1.25	1.19	0.95	1.36	1.03	1.34	1.13	1.26
	4	1.08	1.14	1.23	1.16	1.40	1.13	1.10	1.17	1.20	1.19	1.30	1.18
	5	1.12	1.10	1.29	1.13	1.46	1.12	1.15	1.13	1.27	1.16	1.38	1.15
0.30	8	1.22	1.05	1.43	1.09	1.60	1.09	1.27	1.07	1.43	1.11	1.54	1.11
	10	1.26	1.03	1.50	1.07	1.67	1.08	1.32	1.05	1.51	1.09	1.62	1.10
	1	0.90	1.56	0.99	1.36	1.18	1.22	0.89	1.69	0.93	1.52	1.04	1.35
	2	0.97	1.28	1.09	1.24	1.28	1.18	0.96	1.34	1.04	1.32	1.15	1.25
	4	1.10	1.12	1.26	1.15	1.43	1.13	1.11	1.15	1.22	1.19	1.32	1.17
	5	1.15	1.09	1.32	1.12	1.49	1.11	1.17	1.12	1.29	1.15	1.40	1.15
	8	1.25	1.04	1.47	1.08	1.64	1.09	1.30	1.06	1.45	1.10	1.56	1.11
	10	1.30	1.02	1.53	1.07	1.71	1.07	1.35	1.04	1.53	1.09	1.65	1.09
	1	0.89	1.66	0.94	1.51	1.03	1.35	0.92	1.56	0.96	1.48	1.07	1.32
	2	1.00	1.33	1.08	1.30	1.16	1.24	1.07	1.29	1.13	1.27	1.24	1.21
0.45	4	1.18	1.17	1.30	1.18	1.38	1.15	1.30	1.15	1.40	1.15	1.51	1.13
	5	1.25	1.14	1.38	1.15	1.47	1.13	1.40	1.12	1.51	1.13	1.61	1.11
	8	1.40	1.08	1.57	1.10	1.66	1.09	1.59	1.07	1.74	1.09	1.84	1.08
	10	1.47	1.06	1.66	1.08	1.76	1.08	1.69	1.06	1.85	1.07	1.96	1.06
	1	0.90	1.61	0.96	1.47	1.06	1.32	0.93	1.51	0.99	1.42	1.09	1.30
	2	1.03	1.30	1.10	1.28	1.20	1.22	1.09	1.27	1.18	1.24	1.27	1.20
	4	1.23	1.15	1.34	1.16	1.43	1.14	1.35	1.14	1.46	1.14	1.55	1.12
	5	1.30	1.12	1.43	1.14	1.52	1.12	1.45	1.11	1.57	1.11	1.66	1.10
	8	1.47	1.07	1.63	1.09	1.73	1.08	1.66	1.07	1.81	1.08	1.90	1.07
	10	1.55	1.05	1.72	1.08	1.83	1.07	1.76	1.05	1.92	1.07	2.02	1.06
0.30	1	0.91	1.58	0.97	1.46	1.07	1.31	0.93	1.51	0.99	1.42	1.10	1.29
	2	1.03	1.29	1.11	1.28	1.22	1.21	1.10	1.26	1.18	1.24	1.28	1.20
	4	1.24	1.14	1.35	1.16	1.45	1.13	1.36	1.14	1.47	1.14	1.56	1.12
	5	1.32	1.11	1.44	1.13	1.54	1.11	1.46	1.11	1.58	1.11	1.67	1.10
	8	1.50	1.06	1.65	1.09	1.75	1.08	1.68	1.07		1.08	1.92	1.07

Table 6 (continued)

$\rho_{j,base}$	$\rho_{j,btc}$	M_{2LL}		D_{2LL}		T_{2LL}		M_{3LL}		D_{3LL}		T_{3LL}	
		SI^{US-ELM}	$\frac{SI^{US-ELM}}{SI^{US-NOLM}}$	SI^{US-ELM}	$\frac{SI^{US-ELM}}{SI^{US-NOLM}}$	SI^{US-ELM}	$\frac{SI^{US-ELM}}{SI^{US-NOLM}}$	SI^{US-ELM}	$\frac{SI^{US-ELM}}{SI^{US-NOLM}}$	SI^{US-ELM}	$\frac{SI^{US-ELM}}{SI^{US-NOLM}}$	SI^{US-ELM}	$\frac{SI^{US-ELM}}{SI^{US-NOLM}}$
10		1.58	1.05	1.75	1.08	1.85	1.07	1.78	1.05	1.07 1.94	1.06	2.04	1.06

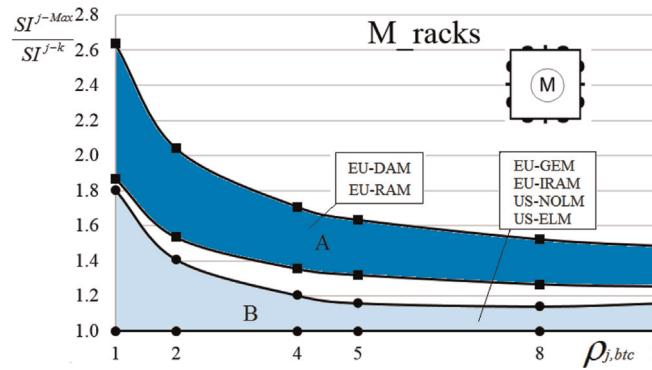
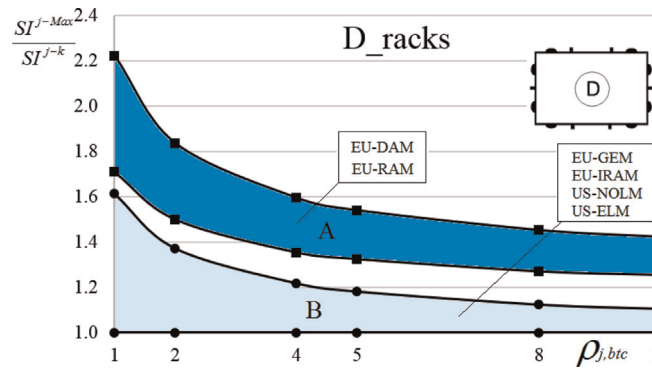
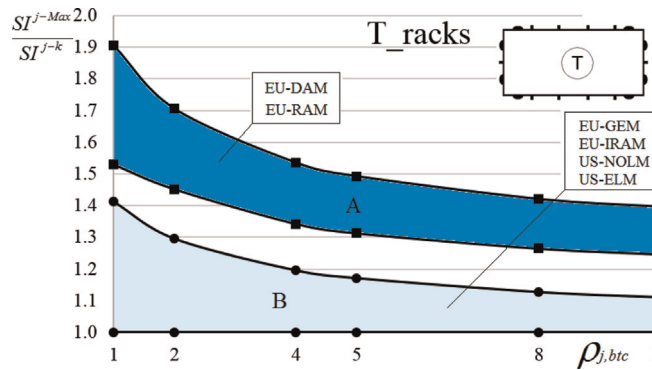
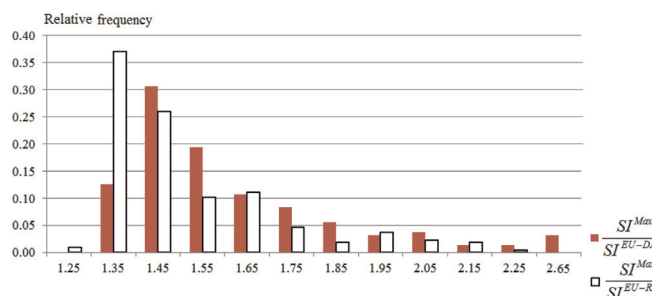
Fig. 15. Comparison between the design approaches in terms of $\frac{SI^{j-Max}}{SI^{j-k}}$ relationship for M_racks.Fig. 16. Comparison between the design approaches in terms of $\frac{SI^{j-Max}}{SI^{j-k}}$ relationship for D_racks.Fig. 17. Comparison between the design approaches in terms of $\frac{SI^{j-Max}}{SI^{j-k}}$ relationship for T_racks.Fig. 18. Distribution of the relative frequency of the $\frac{SI^{j-Max}}{SI^{j-k}}$ relationship.

Table 7
Direct comparison between EU-IRAM, EU-GEM, US-ELM and US-NOLM approach.

$\rho_{j,base}$	$\rho_{j,btc}$	M_{2LL}			D_{2LL}			T_{2LL}			M_{3LL}			D_{3LL}			T_{3LL}		
		$\frac{S_I^{US-ELM}}{S_I^{EU-IRAM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-GEM}}$	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-IRAM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-GEM}}$	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-IRAM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-GEM}}$	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-IRAM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-GEM}}$	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-IRAM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-GEM}}$	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-IRAM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-GEM}}$	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$
0.15	1	1.08	1.13	1.72	1.04	1.09	1.47	1.05	1.09	1.29	1.08	1.12	1.81	1.05	1.09	1.61	1.04	1.08	1.41
	2	1.06	1.09	1.38	1.04	1.08	1.31	1.06	1.09	1.23	1.04	1.08	1.41	1.04	1.08	1.37	1.05	1.08	1.30
	4	1.10	1.08	1.19	1.06	1.08	1.19	1.07	1.09	1.16	1.08	1.08	1.20	1.05	1.08	1.22	1.06	1.08	1.20
	5	1.11	1.08	1.15	1.06	1.08	1.16	1.07	1.09	1.14	1.10	1.08	1.16	1.06	1.08	1.18	1.07	1.08	1.17
	8	1.14	1.08	1.10	1.08	1.09	1.12	1.08	1.09	1.11	1.14	1.08	1.10	1.08	1.09	1.13	1.07	1.09	1.13
	10	1.15	1.08	1.08	1.10	1.09	1.10	1.08	1.09	1.10	1.16	1.09	1.08	1.09	1.09	1.11	1.08	1.09	1.11
	1	1.07	1.12	1.60	1.04	1.09	1.39	1.06	1.09	1.24	1.06	1.11	1.72	1.04	1.09	1.54	1.05	1.09	1.36
	2	1.05	1.08	1.31	1.05	1.08	1.26	1.07	1.09	1.19	1.03	1.07	1.36	1.04	1.08	1.34	1.05	1.08	1.26
	4	1.09	1.08	1.14	1.06	1.09	1.16	1.08	1.09	1.13	1.06	1.07	1.17	1.06	1.09	1.19	1.07	1.09	1.18
	5	1.11	1.08	1.10	1.07	1.09	1.13	1.08	1.09	1.12	1.08	1.07	1.13	1.07	1.09	1.16	1.07	1.09	1.15
0.30	8	1.14	1.09	1.05	1.08	1.09	1.09	1.08	1.09	1.09	1.12	1.08	1.07	1.07	1.09	1.11	1.08	1.09	1.11
	10	1.16	1.09	1.03	1.09	1.09	1.07	1.09	1.09	1.08	1.14	1.08	1.05	1.08	1.09	1.09	1.08	1.09	1.10
	1	1.06	1.11	1.56	1.04	1.09	1.36	1.06	1.09	1.22	1.07	1.12	1.69	1.04	1.09	1.52	1.05	1.09	1.35
	2	1.04	1.09	1.28	1.05	1.09	1.24	1.07	1.09	1.18	1.04	1.08	1.34	1.04	1.08	1.32	1.06	1.09	1.25
	4	1.09	1.08	1.12	1.07	1.09	1.15	1.08	1.09	1.13	1.07	1.08	1.15	1.06	1.08	1.19	1.07	1.09	1.17
	5	1.11	1.09	1.09	1.07	1.09	1.12	1.08	1.09	1.11	1.09	1.09	1.12	1.07	1.09	1.15	1.07	1.09	1.15
	8	1.14	1.09	1.04	1.08	1.09	1.08	1.09	1.09	1.09	1.13	1.09	1.06	1.07	1.09	1.10	1.08	1.09	1.11
	10	1.16	1.09	1.02	1.09	1.09	1.07	1.09	1.10	1.07	1.15	1.09	1.04	1.08	1.09	1.09	1.08	1.09	1.09
$\rho_{j,base}$	$\rho_{j,btc}$	M_{4LL}			D_{4LL}			T_{4LL}			M_{5LL}			D_{5LL}			T_{5LL}		
		$\frac{S_I^{US-ELM}}{S_I^{EU-IRAM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-GEM}}$	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-IRAM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-GEM}}$	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-IRAM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-GEM}}$	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-IRAM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-GEM}}$	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-IRAM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-GEM}}$	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-IRAM}}$	$\frac{S_I^{US-ELM}}{S_I^{EU-GEM}}$	$\frac{S_I^{US-ELM}}{S_I^{US-NOLM}}$
0.15	1	1.05	1.10	1.66	1.04	1.09	1.51	1.05	1.08	1.35	1.04	1.08	1.56	1.02	1.06	1.48	1.05	1.08	1.32
	2	1.04	1.08	1.33	1.05	1.08	1.30	1.06	1.09	1.24	1.05	1.07	1.29	1.04	1.06	1.27	1.06	1.08	1.21
	4	1.07	1.08	1.17	1.07	1.09	1.18	1.07	1.09	1.15	1.07	1.08	1.15	1.06	1.07	1.15	1.08	1.09	1.13
	5	1.09	1.08	1.14	1.07	1.09	1.15	1.08	1.09	1.13	1.08	1.08	1.12	1.07	1.08	1.13	1.08	1.09	1.11
	8	1.13	1.09	1.08	1.08	1.09	1.10	1.08	1.09	1.09	1.12	1.08	1.07	1.08	1.08	1.09	1.09	1.09	1.08
	10	1.14	1.09	1.06	1.08	1.09	1.08	1.08	1.09	1.08	1.13	1.09	1.06	1.09	1.08	1.07	1.09	1.09	1.06
	1	1.04	1.09	1.61	1.04	1.08	1.47	1.05	1.08	1.32	1.03	1.07	1.51	1.04	1.08	1.42	1.05	1.08	1.30
	2	1.04	1.08	1.30	1.05	1.08	1.28	1.06	1.09	1.22	1.03	1.06	1.27	1.06	1.08	1.24	1.07	1.09	1.20
	4	1.06	1.08	1.15	1.07	1.09	1.16	1.08	1.09	1.14	1.05	1.06	1.14	1.08	1.09	1.14	1.08	1.09	1.12
	5	1.08	1.09	1.12	1.08	1.09	1.14	1.08	1.09	1.12	1.06	1.07	1.11	1.08	1.09	1.11	1.08	1.09	1.10
0.30	8	1.12	1.09	1.07	1.08	1.09	1.09	1.09	1.09	1.08	1.08	1.07	1.07	1.09	1.09	1.08	1.09	1.09	1.07
	10	1.13	1.09	1.05	1.09	1.09	1.08	1.09	1.09	1.07	1.10	1.07	1.05	1.09	1.09	1.07	1.09	1.09	1.06
	1	1.05	1.09	1.58	1.04	1.08	1.46	1.05	1.09	1.31	1.04	1.08	1.51	1.03	1.07	1.42	1.06	1.08	1.29
	2	1.04	1.08	1.29	1.05	1.08	1.28	1.06	1.09	1.21	1.05	1.08	1.26	1.05	1.07	1.24	1.07	1.08	1.20
	4	1.06	1.09	1.14	1.07	1.09	1.16	1.08	1.09	1.13	1.07	1.08	1.14	1.07	1.08	1.14	1.08	1.09	1.12
	5	1.08	1.09	1.11	1.08	1.09	1.13	1.08	1.09	1.11	1.08	1.09	1.11	1.08	1.08	1.11	1.09	1.09	1.10
	8	1.11	1.09	1.06	1.08	1.09	1.09	1.09	1.09	1.08	1.10	1.09	1.07	1.09	1.08	1.08	1.09	1.09	1.07
	10	1.13	1.10	1.05	1.08	1.09	1.08	1.09	1.09	1.07	1.11	1.09	1.05	1.09	1.09	1.06	1.09	1.09	1.06

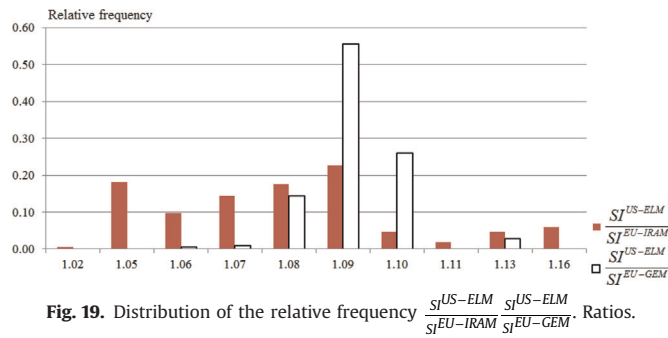


Fig. 19. Distribution of the relative frequency $\frac{SI^{US-ELM}}{SI^{EU-IRAM}} \frac{SI^{US-ELM}}{SI^{EU-GEM}}$ Ratios.

Appendix A. Design example

The scope of the present Appendix is to reproduce the main computations associated with the discussed design approaches, which should be useful as a benchmark for researchers and designers.

The case history has been taken from the numerical analyses described in sub-chapter 2 and refers to the more stressed internal upright of the D_5LL rack (Fig. 1) with $\rho_{j,btc} = 5.0$ and $\rho_{j,base} = 0.3$. The system length of the upright is 1500 mm in the longitudinal (down-aisle) direction and Z-panels in the transversal (cross-aisle) direction having a height of 1250 mm forms the upright frames.

The main cross-section data are reported in Table 1 and reference has to be made to the D_ upright cross-section. The material is S355 steel grade [10] with a yielding strength equal to 355 MPa.

As to design values of axial load and bending moments, reference has to be made to Table A1, where the axial force, and bending moments at the bottom (BOT) and the top (TOP) of the more stressed upright are reported, arising them from second-order structural analyses according to the discussed design approaches.

Table A1

Summary of the key results of the second-order FE analysis (terminology in accordance with EU code).

Method		N_{Ed} [kN]	$M_{y,Ed}^{BOT}$ [kNm]	$M_{y,Ed}^{TOP}$ [kNm]	$M_{z,Ed}^{BOT}$ [kNm]	$M_{z,Ed}^{TOP}$ [kNm]
$F+q$	EU-DAM	276.2	2.36	0.86	0.83	−0.32
$\Phi+\delta$			2.31	0.85	0.80	−0.30
F	EU-RAM		2.07	0.46	0.83	−0.31
	EU-IRAM					
	EU-GEM					
Φ	EU-RAM		2.06	0.45	0.83	−0.31
	EU-IRAM					
	EU-GEM					
US-NOLM		265.61	4.14	0.97	0.69	−0.29
US – ELM			2.91	0.62	0.67	−0.28

A1. The European approaches

In accordance with the requirements of EC3 [7], Young modulus is $E=210,000$ MPa. Furthermore, as to the stability checks, reference is made to an imperfection factor $\alpha=0.34$. In European computations, material safety factors, γ_{Mj} , have been assumed equal to unity, as recommended by the code. Two different approaches to account for imperfection effects have been always considered in the second-order analysis: notional loads and imperfect rack elements.

The value of the global (sway) imperfection is $\phi = 1/357$ according to EN15512.

EU – DAM: two different alternatives have been considered:

- EU-DAM $_{F+q}$: perfect uprights, with notional concentrated and distributed equivalent loads (Fig. 8a);
 - EU-DAM $_{\Phi+\delta}$: curved inclined uprights suitably accounting for sway and bow imperfections (Fig. 8b);
- Resistance checks are required by the EU-DAM approach. The values of the safety index SI^{EU-DAM} corresponding to these design options are:

$$\begin{aligned}
 SI_{F+q}^{EU-DAM} &= \frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed}}{M_{y,Rd}} + \frac{M_{z,Ed}}{M_{z,Rd}} \\
 &= \frac{276.23 \cdot 10^3}{850 \cdot 355} + \frac{2.36 \cdot 10^6}{41597 \cdot 355} + \frac{0.83 \cdot 10^6}{33007 \cdot 355} \\
 &= 0.915 + 0.160 + 0.071 = 1.146
 \end{aligned}$$

$$\begin{aligned}
 SI_{\phi+\delta}^{EU-DAM} &= \frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed}}{M_{y,Rd}} + \frac{M_{z,Ed}}{M_{z,Rd}} \\
 &= \frac{276.19 \cdot 10^3}{850 \cdot 355} + \frac{2.31 \cdot 10^6}{41597 \cdot 355} + \frac{0.80 \cdot 10^6}{33007 \cdot 355} \\
 &= 0.915 + 0.157 + 0.068 = 1.140
 \end{aligned}$$

EU – RAM: two alternatives for imperfections have been considered:

- EU-RAM_F: perfect uprights with a notional concentrated load (Fig. 5a);
- EU-RAM_φ: inclined uprights (Fig. 5b);

In the following, the main contributions of the safety index are evaluated.

$$Critical \text{ buckling load } N_{cr,k} = \frac{\pi^2 E \cdot I_k}{L_{0,k}^2}$$

$$Critical \text{ buckling load } N_{cr,k} = \frac{\pi^2 E \cdot I_k}{L_{0,k}^2}$$

$$N_{cr,y} = \frac{\pi^2 \cdot 210000 \cdot 3466410}{1500^2} = 3193.13 \cdot 10^3 N \quad \left| \quad N_{cr,z} = \frac{\pi^2 \cdot 210000 \cdot 1774529}{1250^2} = 2353.87 \cdot 10^3 N \right.$$

$$\bar{\lambda}_k = \sqrt{\frac{A_{eff} \cdot f_y}{N_{cr,k}}}$$

$$\bar{\lambda}_y = \sqrt{\frac{850 \cdot 355}{3193.13 \cdot 10^3}} = 0.307 \quad \left| \quad \bar{\lambda}_z = \sqrt{\frac{850 \cdot 355}{2353.87 \cdot 10^3}} = 0.358 \right.$$

$$\varphi = 0.5 \cdot [1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2]$$

$$\varphi_y = 0.5 \cdot [1 + 0.34 \cdot (0.31 - 0.2) + 0.31^2] = 0.566 \quad \left| \quad \varphi_z = 0.5 \cdot [1 + 0.34 \cdot (0.36 - 0.2) + 0.36^2] = 0.591 \right.$$

$$\chi_k = \frac{1}{\varphi_k + \sqrt{\varphi_k^2 - \bar{\lambda}_y^2}}$$

$$\chi_y = \frac{1}{0.566 + \sqrt{0.566^2 - 0.307^2}} = 0.961 \quad \left| \quad \chi_z = \frac{1}{0.591 + \sqrt{0.591^2 - 0.358^2}} = 0.942 \right.$$

As to the bending moment k_y and k_z coefficients, for each case, it is necessary to evaluate the equivalent uniform moment factors, $\beta_{M,y}$ and $\beta_{M,z}$ with reference to the effective moment distribution along the system length, about the y- and z-axis, respectively.

EU-RAM_F:

$$\begin{aligned}
 \beta_{M,y} &= 1.8 - 0.7\psi = 1.8 - 0.7 \frac{460000}{2070000} = 1.644 & \left| \quad \beta_{M,z} &= 1.8 - 0.7\psi = 1.8 - 0.7 \frac{-310000}{830000} = 2.061 \right. \\
 \mu_y &= \bar{\lambda}_y (2\beta_{M,y} - 4) = 0.307 \cdot (2 \cdot 1.644 - 4) = -0.221 < 0.9 & \left| \quad \mu_z &= \bar{\lambda}_z (2\beta_{M,z} - 4) = 0.358 \cdot (2 \cdot 2.061 - 4) = 0.044 < 0.9 \right. \\
 k_y &= 1 - \frac{\mu_y N_{Ed}}{\chi_y A_{eff} \cdot f_y} = 1 - \frac{-0.22 \cdot 276220}{0.961 \cdot 850 \cdot 355} = 1.210 > 1 & \left| \quad k_z &= 1 - \frac{\mu_z N_{Ed}}{\chi_z A_{eff} \cdot f_y} = 1 - \frac{0.04 \cdot 276220}{0.942 \cdot 850 \cdot 355} = 0.961 < 1.0 \right. \\
 \rightarrow k_y &= 1.0 & & \\
 SI_F^{EU-RAM} &= \frac{276.22 \cdot 10^3}{0.942 \cdot (850 \cdot 355)} + 1.0 \frac{2.07 \cdot 10^6}{41597 \cdot 355} + 0.961 \frac{0.83 \cdot 10^6}{33007 \cdot 355} = 0.972 + 0.140 + 0.068 = 1.180
 \end{aligned}$$

EU-RAM_φ:

$$\begin{aligned}
 \beta_{M,y} &= 1.8 - 0.7\psi = 1.8 - 0.7 \frac{450000}{2060000} = 1.647 & \left| \quad \beta_{M,z} &= 1.8 - 0.7\psi = 1.8 - 0.7 \frac{-310000}{830000} = 2.061 \right. \\
 \mu_y &= \bar{\lambda}_y (2\beta_{M,y} - 4) = 0.307 \cdot (2 \cdot 1.647 - 4) = -0.217 < 0.9 & \left| \quad \mu_z &= \bar{\lambda}_z (2\beta_{M,z} - 4) = 0.358 \cdot (2 \cdot 2.061 - 4) = 0.044 < 0.9 \right. \\
 k_y &= 1 - \frac{\mu_y N_{Ed}}{\chi_y A_{eff} \cdot f_y} = 1 - \frac{-0.22 \cdot 276220}{0.961 \cdot 850 \cdot 355} = 1.210 > 1 & \left| \quad k_z &= 1 - \frac{\mu_z N_{Ed}}{\chi_z A_{eff} \cdot f_y} = 1 - \frac{0.04 \cdot 276220}{0.942 \cdot 850 \cdot 355} = 0.961 < 1.0 \right. \\
 \rightarrow k_y &= 1.0 & & \\
 SI_{\phi}^{EU-RAM} &= \frac{276.22 \cdot 10^3}{0.942 \cdot (850 \cdot 355)} + 1.0 \frac{2.06 \cdot 10^6}{41597 \cdot 355} + 0.961 \frac{0.83 \cdot 10^6}{33007 \cdot 355} = 0.972 + 0.139 + 0.068 = 1.179
 \end{aligned}$$

EU – IRAM: stability check is based on the effective length evaluated with the use of the factor α_{cr} , obtained from a finite element buckling analysis, giving for the rack the sway buckling multiplier $\alpha_{cr} = 1.78$. Also in this case, two different approaches to model imperfections have been considered:

- EU-IRAM_F: perfect uprights with a notional concentrated load (Fig. 5a);
- EU-IRAM_Φ: inclined uprights (Fig. 5b);

It results $L_{eff} = \sqrt{\frac{\pi^2 EI}{\alpha_{cr} N_{Ed}}} = \sqrt{\frac{\pi^2 210000 \cdot 3466410}{491.78}}$ therefore, $K = 2.55$.

$$\bar{\lambda}_z = \bar{\lambda}_y = \sqrt{\frac{A_{eff} f_y}{N_{cr}}} = \sqrt{\frac{850 \cdot 355}{491.78 \cdot 10^3}} = 0.783$$

$$\begin{aligned} \varphi &= 0.5 \cdot [1 + 0.34 \cdot (\bar{\lambda} - 0.2) + \bar{\lambda}^2] \\ &= 0.5 \cdot [1 + 0.34 \cdot (0.78 - 0.2) + 0.78^2] = 0.902 \end{aligned}$$

$$\chi_{min} = \frac{1}{\varphi + \sqrt{\varphi^2 - \bar{\lambda}_z^2}} = \frac{1}{0.902 + \sqrt{0.902^2 - 0.783^2}} = 0.741$$

- EU-IRAM_F: perfect uprights with a notional concentrated load (fig. 5a);
- EU-IRAM_Φ: inclined uprights (fig. 5b);

It results $L_{eff} = \sqrt{\frac{\pi^2 EI}{\alpha_{cr} N_{Ed}}} = \sqrt{\frac{\pi^2 210000 \cdot 3466410}{491.78}}$ therefore, $K = 2.55$.

$$\bar{\lambda}_z = \bar{\lambda}_y = \sqrt{\frac{A_{eff} \cdot f_y}{N_{cr}}} = \sqrt{\frac{850 \cdot 355}{491.78 \cdot 10^3}} = 0.783$$

$$\varphi = 0.5 \cdot [1 + 0.34 \cdot (\bar{\lambda} - 0.2) + \bar{\lambda}^2] = 0.5 \cdot [1 + 0.34 \cdot (0.78 - 0.2) + 0.78^2] = 0.902$$

$$\chi_{min} = \frac{1}{\varphi + \sqrt{\varphi^2 - \bar{\lambda}_z^2}} = \frac{1}{0.902 + \sqrt{0.902^2 - 0.783^2}} = 0.741$$

EU-IRAM_F:

$$\begin{aligned} \beta_{M,y} &= 1.8 - 0.7\psi = 1.8 - 0.7 \cdot \frac{460000}{2070000} = 1.64 & \beta_{M,z} &= 1.8 - 0.7\psi = 1.8 - 0.7 \cdot \frac{-310000}{830000} = 2.06 \\ \mu_y &= \bar{\lambda}_y (2\beta_{M,y} - 4) = 0.783 \cdot (2 \cdot 1.64 - 4) = -0.564 < 0.9 & \mu_z &= \bar{\lambda}_z (2\beta_{M,z} - 4) = 0.783 \cdot (2 \cdot 2.06 - 4) = 0.09 < 0.9 \\ k_y &= 1 - \frac{\mu_y N_{Ed}}{\chi_y A_{eff} \cdot f_y} = 1 - \frac{-0.564 \cdot 276220}{0.74 \cdot 850 \cdot 355} = 1.68 > 1 & k_z &= 1 - \frac{\mu_z N_{Ed}}{\chi_z A_{eff} \cdot f_y} = 1 - \frac{0.09 \cdot 276220}{0.74 \cdot 850 \cdot 355} = 0.889 < 1.0 \\ &\rightarrow k_y = 1.0 & & \end{aligned}$$

$$SI_F^{EU-IRAM} = \frac{276.22 \cdot 10^3}{0.736 \cdot (850 \cdot 355)} + 1.0 \cdot \frac{2.07 \cdot 10^6}{41597 \cdot 355} + 0.889 \cdot \frac{0.83 \cdot 10^6}{33007 \cdot 355} = 1.245 + 0.140 + 0.063 = 1.448$$

EU-IRAM_Φ:

$$\begin{aligned} \beta_{M,y} &= 1.8 - 0.7\psi = 1.8 - 0.7 \cdot \frac{450000}{2060000} = 1.65 & \beta_{M,z} &= 1.8 - 0.7\psi = 1.8 - 0.7 \cdot \frac{-310000}{830000} = 2.06 \\ \mu_y &= \bar{\lambda}_y (2\beta_{M,y} - 4) = 0.783 \cdot (2 \cdot 1.65 - 4) = -0.548 < 0.9 & \mu_z &= \bar{\lambda}_z (2\beta_{M,z} - 4) = 0.783 \cdot (2 \cdot 2.06 - 4) = 0.09 < 0.9 \\ k_y &= 1 - \frac{\mu_y N_{Ed}}{\chi_y A_{eff} \cdot f_y} = 1 - \frac{-0.548 \cdot 276220}{0.74 \cdot 850 \cdot 355} = 1.67 > 1 & k_z &= 1 - \frac{\mu_z N_{Ed}}{\chi_z A_{eff} \cdot f_y} = 1 - \frac{0.09 \cdot 276220}{0.74 \cdot 850 \cdot 355} = 0.889 < 1.0 \\ &\rightarrow k_y = 1.0 & & \end{aligned}$$

$$SI_\Phi^{EU-IRAM} = \frac{276.22 \cdot 10^3}{0.736 \cdot (850 \cdot 355)} + 1.0 \cdot \frac{2.06 \cdot 10^6}{41597 \cdot 355} + 0.889 \cdot \frac{0.83 \cdot 10^6}{33007 \cdot 355} = 1.245 + 0.140 + 0.063 = 1.448$$

EU – GEM: only the effects of global (sway) imperfection are accounted for in this case, two different possibilities have been considered:

EU-GEM_F (fig. 5a): perfect uprights with notional loads $\alpha_{ult,k(F)} = \frac{1}{\frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed}}{M_{y,Rd}} + \frac{M_{z,Ed}}{M_{z,Rd}}} = 0.888$ $\bar{\lambda}_{op(F)} = \sqrt{\frac{\alpha_{ult,k(F)}}{\alpha_{cr}}} = \sqrt{\frac{0.888}{1.78}} = 0.706$ $\chi_{op(F)} = \frac{1}{0.835 + \sqrt{0.835^2 - 0.706^2}} = 0.781$ $SI_F^{EU-GEM} = \frac{1}{\chi_{op} \cdot \alpha_{ult,k(F)}} = 1.442$	EU-GEM_Φ (fig. 5b): inclined uprights $\alpha_{ult,k(\Phi)} = \frac{1}{\frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed}}{M_{y,Rd}} + \frac{M_{z,Ed}}{M_{z,Rd}}} = 0.889$ $\bar{\lambda}_{op(\Phi)} = \sqrt{\frac{\alpha_{ult,k(\Phi)}}{\alpha_{cr}}} = \sqrt{\frac{0.889}{1.78}} = 0.707$ $\chi_{op(\Phi)} = \frac{1}{\varphi_{op} + \sqrt{\varphi_{op}^2 - \bar{\lambda}_{op}^2}} = 0.781$ $SI_{\Phi}^{EU-GEM} = \frac{1}{\chi_{op} \cdot \alpha_{ult,k(\Phi)}} = 1.440$
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A2. The United States approaches

In accordance with ANSI/AISC-360, Young modulus $E=199,950$ MPa is assumed. Preliminarily to the computation, the possibility of lateral-flexural buckling has been evaluated. In particular, the limit length L_u results:

$$L_u = \frac{0.36C_b\pi}{F_y S_f} \cdot \sqrt{EI_y GJ} = \frac{0.36 \cdot 1 \cdot \pi}{355 \cdot 44728} \cdot \sqrt{200000 \cdot 1775000 \cdot 76923 \cdot 3628100} \approx 22400 \text{ mm}$$

The values of the effective cross section parameters are:

$$Q_N^{US} = Q_N^{EU} \rightarrow Q_M^{US} = 0.5 + \frac{Q_N^{EU}}{2} = 0.925$$

The value of the global (sway) imperfection is $\Phi = 1/240$ according to RMI specification.

US – NOLM: check has to be done assuming an effective length factor with $K=1$, and as a consequence lateral torsional buckling can be neglected.

Reduction of the yield strength for compression:

$$F_{el,x} = \frac{\pi^2 E}{(KL/r_x)^2} = \frac{1973920.88}{(1 \cdot 1500/58.9)^2} = 3044 \text{ MPa} \quad \left| \quad F_{el,y} = \frac{\pi^2 E}{(KL/r_y)^2} = \frac{1973920.88}{(1 \cdot 1250/42.1)^2} = 2239 \text{ MPa} \right.$$

$$F_{el} = \min\{F_{el,x}; F_{el,y}\} = 2239.1 \text{ MPa}$$

$$\lambda_c = \sqrt{\frac{F_y}{F_{el}}} = \sqrt{\frac{355}{2239}} = 0.398 < 1.5$$

$$F_n = \left[0.658 \frac{F_y}{F_{el}} \right] F_y = \left[0.658 \frac{355}{2239.19} \right] 355 = 332 \text{ MPa}$$

$$A_e = \left(1 - (1 - Q_{US}^N) \left(\frac{F_n}{F_y} \right)^{Q_{US}^N} \right) A_{net} = \left(1 - (1 - 0.85) \left(\frac{332}{355} \right)^{0.85} \right) 1000 = 858 \text{ mm}^2$$

$$P_n = A_e \cdot F_n = 332 \cdot 858 = 285.11 \text{ kN}$$

Safety index $SI^{US-NOLM}$ is:

$$SI^{US-NOLM} = \frac{P_c}{\phi_c P_n} + \frac{M_{rx}}{\phi_b M_{nx}} + \frac{M_{ry}}{\phi_b M_{ny}} = \frac{265.61 \cdot 10^3}{0.9 \cdot 285109} + \frac{4.14}{0.9 \cdot 14.69} + \frac{0.69}{0.9 \cdot 11.65} = 1.035 + 0.313 + 0.066 = 1.415$$

$$\lambda_c = \sqrt{\frac{F_y}{F_{el}}} = \sqrt{\frac{355}{2239}} = 0.398 < 1.5$$

$$F_n = [0.658 \frac{F_y}{F_{el}}] F_y = [0.658 \frac{355}{2239.19}] 355 = 332 \text{ MPa}$$

$$A_e = (1 - (1 - Q_{US}^N) (\frac{F_n}{F_y})^{Q_{US}^N}) A_{net} = (1 - (1 - 0.85) (\frac{332}{355})^{0.85}) 1000$$

$$= 858 \text{ mm}^2$$

$$P_n = A_e \cdot F_n = 332 \cdot 858 = 285.11 \text{ kN}$$

Safety index $SI^{US-NOLM}$ is:

$$SI^{US-NOLM} = \frac{P_r}{\phi_c P_n} + \frac{M_{rx}}{\phi_b M_{nx}} + \frac{M_{ry}}{\phi_b M_{ny}} = \frac{265.61 \cdot 10^3}{0.9 \cdot 285.109} + \frac{4.14}{0.9 \cdot 14.69} + \frac{0.69}{0.9 \cdot 11.65}$$

$$= 1.035 + 0.313 + 0.066 = 1.415$$

US – ELM: upright check has to be performed with the value of effective length derived from finite element buckling analysis ($K=2.53$), and consequently, in this case lateral torsional buckling can be neglected.

$$F_{el} = \frac{\pi^2 E}{(KL/r_x)^2} = \frac{1973920.88}{(2.53 \cdot 1500 / 58.9)^2} = 475.49 \text{ MPa}$$

$$\lambda_c = \sqrt{\frac{F_y}{F_{el}}} = \sqrt{\frac{355}{475.49}} = 0.864 < 1.5$$

$$F_n = [0.658 \frac{F_y}{F_{el}}] F_y = [0.658 \frac{355}{475.5}] 355 = 259.7 \text{ MPa}$$

$$A_e = (1 - (1 - Q_{US}^N) (\frac{F_n}{F_y})^{Q_{US}^N}) A_{net} = (1 - (1 - 0.85) (\frac{259.7}{355})^{0.85}) 1000$$

$$= 885 \text{ mm}^2$$

$$P_n = A_e \cdot F_n = 259.7 \cdot 885 = 229.84 \text{ kN}$$

Safety index SI^{US-ELM} is:

$$SI^{US-ELM} = \frac{P_r}{\phi_c P_n} + \frac{M_{rx}}{\phi_b M_{nx}} + \frac{M_{ry}}{\phi_b M_{ny}} = \frac{265.61 \cdot 10^3}{0.9 \cdot 229.834} + \frac{2.91}{0.9 \cdot 14.69} + \frac{0.67}{0.9 \cdot 11.65}$$

$$= 1.284 + 0.220 + 0.064 = 1.568$$

A3. Results comparison

At first, it is worth mentioning that all these SI values are greater than unity but the scope of the present appendix as well as of the companion paper is to propose a comparison independent from the acceptability or not of the verification checks from a designer's point of view.

Table A2 represents the final data associated with all EU and US approaches in terms of safety index (SI), reporting also the terms (according to the EU terminology) related to the axial load (SI_N) and bending moments along the y- and z-axis (SI_{My} and SI_{Mz} , respectively). In general, it can be noted that the contribution due to bending moments is quite limited with respect to the one associated with axial load, especially for bending moments (M_z) along the cross-aisle direction.

A quite wide dispersion of the results can be noted with reference to the EU approaches: the safety index associated with the EU-DAM and EU-RAM (comprised between 1.146 and 1.180) are significantly lower than the values (1.44) associated with both EU-IRAM (1.448) and EU-GEM (1.442). As to the US approaches, the safety index associated with the US-NOLM method is practically equal to the SI associated with EU-IRAM and EU-GEM ones. Furthermore, the US-ELM approach provides a more conservative evaluation of the member

Table A2
Summary of the key verification results (terminology in accordance with EU code).

Method		SI_N	SI_{My}	SI_{Mz}	SI	SI^{Max}/SI^{j-k}
EU-DAM	$F+q$	0.915	0.160	0.071	1.146	1.36
	$\Phi+\delta$	0.915	0.157	0.068	1.140	1.38
EU-RAM	F	0.972	0.140	0.068	1.180	1.33
	Φ	0.972	0.139	0.068	1.179	1.33
EU-IRAM	F	1.245	0.140	0.063	1.448	1.09
	Φ	1.245	0.140	0.063	1.448	1.09
EU-GEM	F	—	—	—	1.442	1.09
	Φ	—	—	—	1.440	1.09
US – NOLM		1.035	0.313	0.066	1.415	1.10
US – ELM		1.284	0.202	0.064	1.568	1.00

performance, being the safety index equal to 1.568. Furthermore, it should be noted that excluding the EU-DAM and the EU-RAM approaches the differences between the SI associated with the other methods are however lower than 11%.

Appendix B. List of symbols

Latin upper case letters

A=gross cross-section area.
 A_{eff} =effective cross-section area.
 AISC=American Institute of Steel Construction.
 ANSI=American National Standards Institute.
 DAM=Direct Analysis Method.
 E =modulus of elasticity of steel.
 E_d =design value.
 EC3=EN 1993-1-1 Eurocode 3 "Design of Steel Structures".
 ELM=Effective Length Method.
 RMI=Rack Manufacturers Institute.
 EU=Europe, European.
 F_n =critical stress.
 F_{el} =elastic buckling stress.
 F_y =yielding strength.
 $F+q$ =notional equivalent loads for simulated local and global imperfections.
 G =shear material modulus.
 GEM=General Method.
 LL=load levels.
 L =member length.
 L_{eff} =effective buckling length.
 L_u =member length for flexural buckling instability.
 I_t =Saint-Venant torsion constant.
 I_w =warping constant.
 I_y, I_z =second moment of area.
 K =effective length factor.
 IRAM=improved rigorous analysis method.
 $M_{Ed}, M_{y,Ed}, M_{z,Ed}$ =design bending moment.
 $M_{y,F+q}, M_{z,F+q}$ =bending moment associated with the (F+q) approach.
 $M_{y,F}, M_{z,F}$ =bending moment associated with the (F) approach.
 $M_{y,\Phi+\delta}, M_{z,\Phi+\delta}$ =bending moment associated with the ($\Phi+\delta$) approach.
 $M_{y,\Phi}, M_{z,\Phi}$ =bending moment associated with the (Φ) approach.
 $M_{j,Ed, \min}$ or $M_{j,Ed, \max}$ =minimum or maximum design bending moment.
 $M_n, M_{y,n}, M_{z,n}$ =nominal bending resistance.
 M_{Rk} =characteristic bending resistance.
 N_{cr} =critical load for the i-member.
 N, N_{Ed} =member axial load.
 N_{Rk} =characteristic axial resistance.
 $N_{b,Rd}$ =axial stability resistance.
 P_c =design axial strength.
 P_n =nominal resistance strength for compression.
 $Q, Q^N, Q_{EU}^N, Q_{US}^N$ =reduction factor for axial load.
 Q_{EU}^{Mz}, Q_{EU}^{My} =reduction factor bending moment.
 RAM=Rigorous Analysis Method.
 R_d =resistance.
 $S_{j,btc}, S_{j,base}$ =stiffness of connection.
 $SI_{j,btc}^{EC3-LB}$ =lower bound of EC3.
 $SI_{j,btc}^{EC3-UB}$ =upper bound of EC3.
 $SI_{j,base}^{EC3-UB}$ =upper bound of EC3 for base-plate connections.
 SI^{k-k} =safety index associated with the j-code and the k- design approach.
 SI, SI^{EU}, SI^{US} =design safety index.
 US=United State of America.
 $W_{eff}, W_{eff,y}, W_{eff,z}$ =effective cross-section modulus.

Latin lower case letters

 e_0 =maximum out-of-straightness defect (bow) imperfection.
 q_0 =distributed load simulating the out-of-straightness defect (bow) imperfection.
 e =eccentricity.
 h =interstorey height.
 k_j, k_z, k_y =bending interaction factor.
 Max=maximum value.

Min=minimum value.

f_y =specified minimum yield stress strength.

Greek case letters

α =imperfection coefficient associated with the relevant buckling curve.

α_{cr} =buckling overall frame multiplier obtained via a finite element buckling analysis.

$\alpha_{ult,k}$ =minimum load multiplier evaluated with reference to the cross-section resistance.

β_{Mj} =bending moment distribution coefficient.

Φ =global imperfection displacement.

δ =bow imperfection displacements.

Δ =sway imperfection displacement.

ψ =gradient moment coefficient.

$\bar{\lambda}_{op}$ =relative slenderness of the whole structure.

$\bar{\lambda}_C$ =slenderness factor.

μ_j =non-dimensional term for beam-column verification check.

$\rho_{j,btc}$ =parameter to define the elastic rotational stiffness of beam-to-column joints.

$\rho_{j,base}$ =parameter to define the elastic rotational stiffness of base-plate joints.

χ =reduction factor for the relative buckling curve.

χ_{LT} =reduction factor due to lateral buckling.

χ_{op} =buckling reduction factor referred to the overall structural system.

$\gamma_M=\gamma_{M1}$ =material safety factor.

Table B1

Comparison between EU and US codes terminology.

EU	Term	US
N_{Ed}	Axial force demand	P_r
$N_{b,Rd}$	Design axial strength	P_n
$M_{y,Ed}, M_{z,Ed}$	Required flexural strength about centroidal axes.	M_{rx}, M_{ry}
$M_{y,Rd}, M_{z,Rd}$	Design flexural strength about centroidal axes.	M_{cx}, M_{cy}
N_{cr}	Elastic critical buckling load	P_e
W_{eff}	Elastic section modulus of effective cross-section	S_e
I_y, I_z	Second moment of Area about centroidal axes	I_x, I_y
I_t	Saint-Venant torsion constant	J
I_w	Torsional warping constant of cross-section	C_w
i_y, i_z	Radius of gyration about symmetry centroidal axes.	r_x, r_y
f_y	Specified minimum yield stress strength	F_y
$y-y, z-z$	Cross-section axes	$x-x, y-y$

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