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**APPLICAZIONE DEL “METODO GENERALE” DELLA EN  
1993-1-1 6.3.4 PER LA PROGETTAZIONE DELL'INSTABILITÀ  
DI MODELLI STRUTTURALI GLOBALI**

**APPLICATION OF THE “GENERAL METHOD” OF EN 1993-1-1  
6.3.4 FOR THE BUCKLING DESIGN OF GLOBAL  
STRUCTURAL MODELS**

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**ABSTRACT**

The practical use of the “General method” of EN 1993-1-1 6.3.4 for the buckling design of global structural models is still a challenging issue requiring several problems to solve. In this paper we propose a fully developed methodology presenting solutions for the application topics such as the suitable FE model, specific modeling issues to capture the true 3D behavior of the members and the whole model and the final evaluation of the design parameters. The presented methodology consistently uses a unique model for the evaluation of all analysis and design parameters and results and yields a fully automatic design process controlled solely by the properly created structural model.

**SOMMARIO**

L'uso pratico del “Metodo generale” della EN 1993-1-1 6.3.4 per la progettazione dell'instabilità di modelli strutturali globali è ancora una questione impegnativa che richiede diversi problemi da risolvere. In questo documento proponiamo una metodologia completamente sviluppata che presenta soluzioni per gli argomenti applicativi come il modello FE adatto, problemi di modellazione specifici per catturare il vero comportamento 3D dei membri e dell'intero modello e la valutazione finale dei parametri di progettazione. La metodologia presentata utilizza costantemente un modello unico per la valutazione di tutti i parametri e risultati di analisi e progettazione e produce un processo di progettazione completamente automatico controllato esclusivamente dal modello strutturale opportunamente creato.

## 1 INTRODUCTION

The “General method” for the buckling design of steel structural members is defined in Chapter 6.3.4 of the EN 1993-1-1 **Errore. L'origine riferimento non è stata trovata..** The name of the method is coming from the basic idea of generalizing the parameters used for the traditional buckling design. The purpose of this generalization is to handle the complex loading situations together instead of using the usual separated load effects. Accordingly, the method uses load effect amplifiers instead of direct use of internal forces and bending moments. Table 1 shows the out-of-plane buckling design parameters of members subjected to compressive force and bending moment in case of the traditional design method and the “General method”.

**Table 1.** Parameters of member buckling design methods

| Design parameters                   | Traditional method                           |                                                      | General method                                                 |
|-------------------------------------|----------------------------------------------|------------------------------------------------------|----------------------------------------------------------------|
| Load effect                         | Compression                                  | Bending moment                                       | Complex                                                        |
| Buckling mode                       | Flexural buckling                            | Lateral-torsional buckling                           | Compound                                                       |
| Elastic critical parameters         | $N_{cr}$                                     | $M_{cr}$                                             | $\alpha_{cr}$                                                  |
| Cross-section resistance parameters | $Af_y$                                       | $W_y f_y$                                            | $\alpha_{ult}$                                                 |
| Slenderness                         | $\bar{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}}$ | $\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$ | $\bar{\lambda}_{op} = \sqrt{\frac{\alpha_{ult}}{\alpha_{cr}}}$ |
| Reduction factor                    | $\chi$                                       | $\chi_{LT}$                                          | $\chi_{op}$                                                    |
| Buckling resistance                 | $\chi Af_y$                                  | $\chi_{LT} W_y f_y$                                  | $\chi_{op} \alpha_{ult}$                                       |

In Table 1  $\alpha_{cr}$  is the complex load effect amplifier to reach the elastic critical state of the member (interaction of flexural and lateral-torsional buckling) and  $\alpha_{ult}$  is the complex load effect amplifier to reach the characteristic resistance of the most critical cross section. Based on these parameters one single out-of-plane buckling slenderness is calculated ( $\bar{\lambda}_{op}$ ) and the buckling resistance is represented by a reduced load effect amplifier ( $\chi_{op} \alpha_{ult}$ ). More detailed description and application of the method is given in **Errore. L'origine riferimento non è stata trovata..**

## 2 APPLICATION ISSUES TO GLOBAL STRUCTURAL MODELS

The most promising potential of the “General method” is the possibility for automatic buckling design of members in any global 3D structural model. It can be done because there is no need to isolate members, separate the pure buckling modes and define the usual buckling parameters (effective length factor, unconstrained length, moment gradient factor, equivalent support conditions etc.) but the method considers the complex system of forces in the member and evaluates directly the appropriate compound buckling modes based on the global structural model. The most efficient and practical way is to do so is the *linear buckling analysis (LBA)* of the whole model. This is a much more consistent method compared to the traditional one, because all the necessary buckling parameters are evaluated on the very same structural model where the in-plane deformations and member forces are calculated. In the traditional method the buckling parameters are always independently defined by either manual input or calculated on a different model. There are however some issues which should be considered if the global model based LBA is combined with the “General method”:

- 1) The FE model of the global structure used for the LBA should have the possibility to cover the calculation of all the global buckling mode types (flexural, torsional, flexural-torsional, lateral-torsional buckling or any interaction) considering any (symmetric or

- asymmetric) cross-section, any support, restraint, connection or load condition (with any eccentricity)
- 2) The realistic continuity of the out-of-plane deformations (out-of-plane rotation, torsion and warping) between the members considering the connection mechanics should be taken into account (beside the continuity of the in-plane deformations like the connection stiffness) because it can significantly influence the boundaries for the out-of-plane buckling of the members
  - 3) For the generalized member slenderness, a certain  $\alpha_{cr}$  value is needed for each member selected from the buckling modes calculated on the global structural model. Accordingly for each member the most critical global buckling mode should be selected.

In the further sections these issues are briefly discussed and the complete solution implemented into Consteel software **Errore. L'origine riferimento non è stata trovata.** is presented through an example.

## 2.1 The FE model

To capture the true 3D behavior of spatial structural models composed of steel members for the evaluation of the global buckling modes described previously the usual 6 DOF beam element is not adequate. The solution would be the application of shell FE for the members but it is impossible to do for complete structural models. The mechanically appropriate and still practical choice is the 7 DOF beam element based on the Vlasov theory completely adequate for the true global 3D behavior of steel members composed of relatively thin-walled open cross-sections. The deformation can be described by three displacements ( $u, v, w$ ) and three rotations ( $\theta_x, \theta_y, \theta_z$ ) of the end nodes of the element Fig. 1. In order to take the complete torsion and warping deformations into consideration, the 7<sup>th</sup> displacement per nodes was introduced as ‘mathematical displacement’:

$$\theta'_x = \frac{d\theta_x}{dx} \quad (1)$$

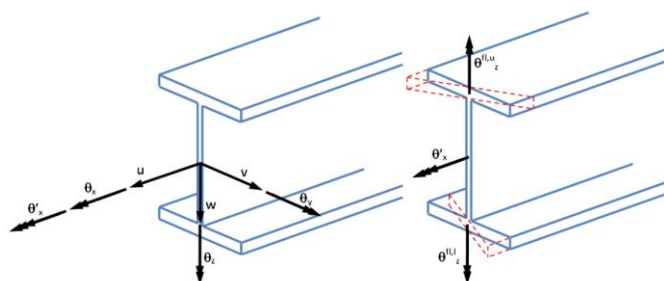
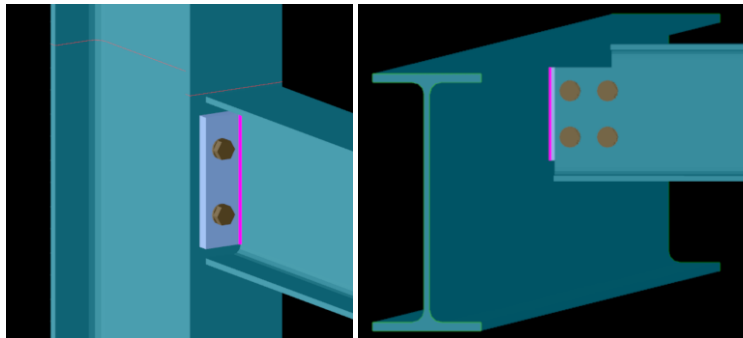


Fig. 1. The 7 deformations degree of freedom

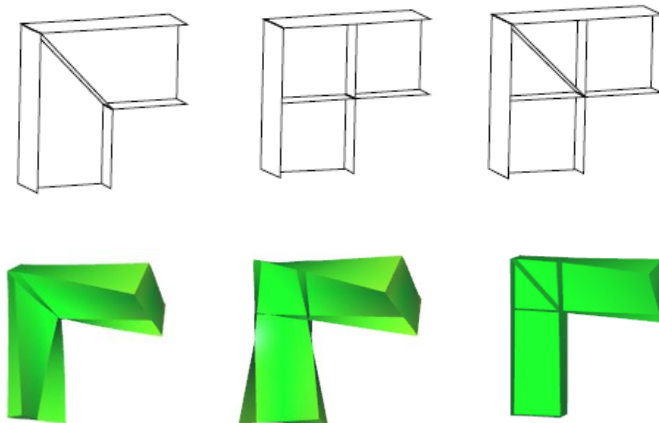
The 7<sup>th</sup> DOF is responsible for several additional mechanical features of the element in the first order and geometric stiffness as well compared to the traditional 6 DOF element. These features make the element capable of calculating all the necessary global buckling mode types considering any cross-section, any support, restraint, connection or load condition detailed in the previous section. The interested reader may find additional information about the performance of this element in the literature including issues for the influence of load, restraint eccentricities, special support conditions (like continuous sheeting (shear field) support on a beam), first and second order deformations and buckling modes [6].

## 2.2 The out-of-plane deformation continuity between members

It is not enough to use a proper FE model for the members of a global structural model. The true 3D behavior captured on the member level should be extended to structure level which requires the accurate transfer of this behavior between the members. For the correct evaluation of the buckling modes the most important is the transfer of the expanded out-of-plane deformations of the 7 DOF beam element: the out-of-plane rotation, complete torsion and warping. In some cases it can be modeled by appropriate releases at the members ends like in the connection configurations in Fig. 2 where rotations and the warping deformations of the connected beam members should be released (simple plate of fin plate connections).



**Fig. 2.** Pinned and warping free connections (Consteel models)



**Fig. 3.** Continuous frame corner behavior: diagonal, box and box-diagonal

The situation can be more complicated if the members are connected by continuous connections (for instance welded or moment end-plate connections) where even the in-plane stiffness is usually not evident (however at least controlled by the Eurocode at some level), but the out-of-plane deformation continuity can be very different influencing the proper LBA calculations. The problem can be best illustrated by the deformations of a simple frame corner with different stiffening solutions (Fig. 3). The three different stiffener configurations are referred as to diagonal, box and box-

diagonal respectively. The deformations of the corners are generated by a simple torsional displacement on the beam ends and calculated by shell elements. It can be well seen that the behavior of the different corners is completely different: the diagonal corner transfers the rotation and warping deformations in opposite direction as the box corner, and the box-diagonal corner practically restraints all torsional and warping deformations.

The solution for this problem is the application of special constraints at the member ends for the transfer of out-of-plane effects following the realistic mechanics of the corner behavior. A simple but very efficient and practically accurate method is developed and implemented in Consteel providing solution for the most widely used frame corner configurations. The most important and influential issue is how to transfer the 7<sup>th</sup> DOF (having impact on the transfer of the complete out-of-plane effects but especially to the warping). Accordingly, the 7<sup>th</sup> DOF is transferred separately from the others through a special constraint equation depending on the actual connection configuration:

$$K_1 \theta'_{xA} + K_2 \theta'_{xB} = 0 \quad (2)$$

where  $\theta'_{xA}$  and  $\theta'_{xB}$  are the warping displacements of the connected member ends  
 $K_1$  and  $K_2$  are connection dependent transfer parameters according to Table 2.

**Table 2.** The warping transfer at continuous member connections

| Connection types                                                                                    | Warping transfer  | Transfer parameters               |
|-----------------------------------------------------------------------------------------------------|-------------------|-----------------------------------|
| Bolted or welded connections where the members are connected through a diagonally placed connection | Full and direct   | $K_1 = 1, K_2 = -1$               |
| Bolted or welded connections with box-type stiffeners                                               | Full and indirect | $K_1 = 1, K_2 = 1$                |
| Bolted or welded connections with box and diagonal type stiffeners                                  | Warping rigid     | $\theta'_{xA} = \theta'_{xB} = 0$ |

In Consteel [5] we have further developed a superelement solution for even more sophisticated member connections, the interested reader may find the results in the literature [7]

### 2.3 Selection of the most critical buckling mode

Having calculated enough number of accurate buckling mode on the global structural model the next problem is how to find the most critical one to a certain member for evaluating its generalized member slenderness. In the case of a large complex 3D structural model with several load combinations and a great amount of different but relevant buckling modes this selection is not evident, and the manual selection would usually be impossible. To offer an automatic and mechanically reliable solution for this problem a special *Mode Relevance Factor (MRF)* is developed which shows the relative relevancy of a given buckling mode for each member of the structural model. This factor is evaluated using a special *Buckling Sensitivity Analysis (BSA)* as a post-process calculation on the LBA results [8]. As a fundamental measure the *BSA* uses the internal deformation energy generated by the actual  $i$ -th buckling mode on the  $k$ -th member:

$$E_i^k = \frac{1}{2} U_i^T K_S^k U_i \quad (3)$$

where  $U_i^T$  is the  $i$ -th buckling mode shape vector

$K_S^k$  is the global stiffness matrix with complied values only for the  $k$ -th member

Using this measure the *MRF* can be defined indicating for the  $i$ -th buckling mode what the relevant (critical) members ( $k$ ) are. The basic assumption for this factor is that each buckling mode has one (or more) specific member(s) which is (are) the most critical and all the members are compared to this one to assess the contribution to the buckling:

$$MRF_i^k = 100 \frac{E_i^k}{\text{Max}[E_i^k]_k} [\%] \quad (4)$$

For the most critical member this factor always takes 100%, and the more critical a member the closer is the *MRF* to 100%. This factor can provide a qualitative help for the automatic selection of the relevant buckling mode for the buckling design of members in the complex 3D model.

### 3 THE PROPOSED IMPLEMENTATION METHOD

Based on the introduced problems and solutions the following methodology is proposed for the implementation of an automatic buckling design procedure using the “General method” combined with the global model based LBA:

(1) *Preparation of the usual structural model*

Modeling the members, connectivities, supports and loads in the usual way for the calculation of deformations, member forces and strength design

(2) *Supplementary modeling*

Implementing all additional modeling features necessary for the evaluation of the true 3D behavior of members and the whole structural model: applying accurately the eccentricities of objects, handling the out-of-plane continuity between members (considering the effects of the 7<sup>th</sup> DOF) etc. The result is a consistent structural model ready for the analysis of both in-plane and out-of-plane deformations, member forces and buckling modes.

(3) *Structural analysis*

Performing three basic analysis types on the same global model (according to the categorization of preEN 1993-1-14):

- Elastic first order analysis – LA
- Elastic complete second order analysis with or without geometrical imperfections – GNA (or GNIA)
- Linear elastic buckling analysis with enough number of buckling modes (it is usually adequate to consider buckling modes with  $\alpha_{cr} < 15$ ) – LBA

(4) *Buckling design pre-process*

Calculating the *MRF* for each member in each buckling mode and assign the certain buckling mode to a member with minimum  $\alpha_{cr}$  among the buckling modes where the *MRF* of the member is greater than a limit value ( $\delta$ ):

$$\text{Member}^k \rightarrow \alpha_{cr} \text{ where } \alpha_{cr} = \text{Min}[\{\alpha_{cr,i}\} \in MRF_i^k > \delta] \quad (5)$$

This limit value can be taken between 10-20% based on our practical studies.

(5) *Check of member buckling by the “General method”*

Calculation of the generalized slenderness for each member using the selected  $\alpha_{cr}$  value and evaluating the buckling reduction factor and the design check according to the procedure defined in EN 1993-1-1 6.3.4.

Important to see two features of the methodology above which makes it very unique compared to the traditional buckling design methods:

- *Consistency* – all the mechanical analysis and standard parameter calculations are based on the same and unique structural model including all information for the complete 3D behavior of the structure
- *Adaptivity* – the steps (3-5) of the methodology contain fully automatic calculations without the need of any additional design input, accordingly after modifying anything on the original structural model the whole process can be automatically performed making the design process very fast.

#### 4 ILLUSTRATIVE EXAMPLE

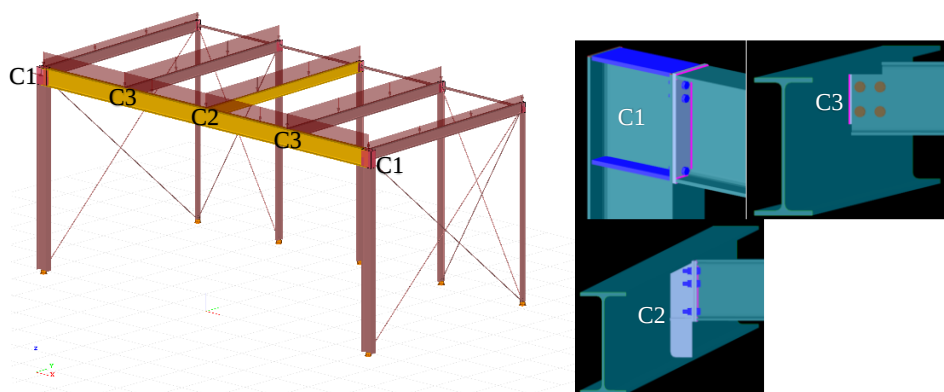
An example calculated by Consteel software [5] is presented briefly to demonstrate mainly the consistency and the adaptivity of the methodology. The example model is not specified fully here the interested reader may contact the author for the full model and result description.

(1) *Preparation of the usual structural model (Fig. 4.)*

Simple model is created with pinned columns, main girder and secondary beams with I sections, the line loads placed eccentrically acting on the top of the beams

(2) *Supplementary modeling (Fig. 4.)*

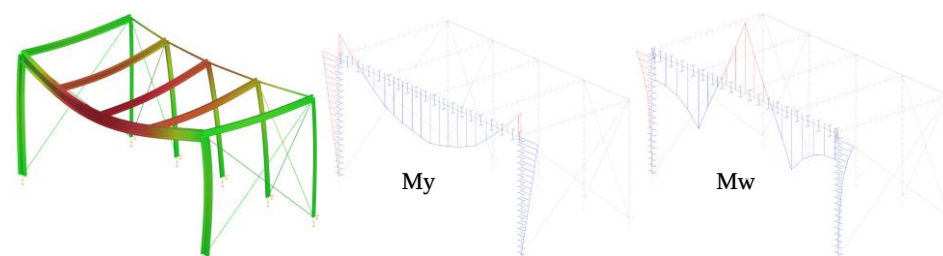
The main girder connected to the columns by box type connection (with full and direct warping transfer), the middle beam connected to the main girder eccentrically and with a stiffened end-plate connection (continuous out-of-plane effect transfer) while the others with fin-plate connections (pinned and warping free transfer)



**Fig. 4.** The structural model and its connections considered

(3) *Structural analysis*

The analysis results – among the usual in-plane deformations and bending moments – shows considerable out-of-plane effects: torsion and warping moment in the main frame, see Fig. 5. 38 different buckling modes are calculated to reach the limit of  $\alpha_{cr} = 15$



**Fig. 5.** The deformation, in-plane bending and warping moments

(4) *Buckling design pre-process*

The results of the BSA and the MRF based selection of the relevant buckling mode and  $\alpha_{cr}$  for each member is shown on Fig. 6.

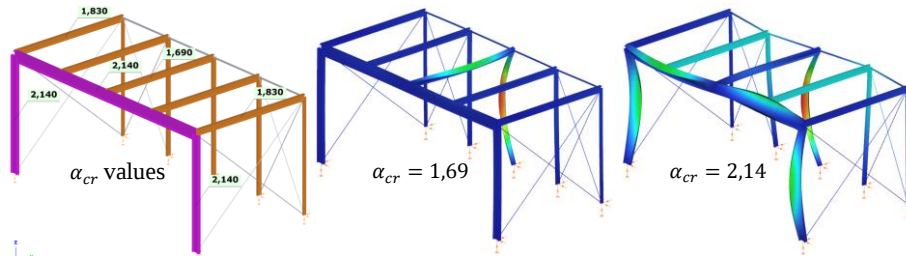


Fig. 6. The BSA and the buckling modes

(5) Check of member buckling by the “General method”

On Fig. 7. one can see the automatically calculated generalized out-of-plane member slenderness values ( $\bar{\lambda}_{op}$ ) and the final utilization of the members.

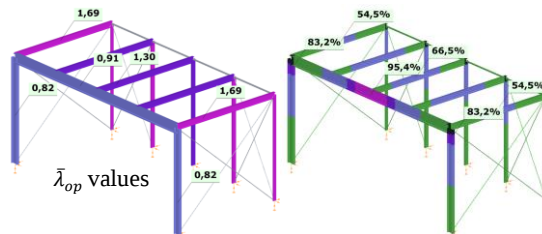


Fig. 7. The member slenderness and the final utilization values

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## KEYWORDS

General method, global structural model, linear buckling analysis, 7 DOF beam element, out-of-plane effects, warping, buckling sensitivity